विघ्न विचारत भीरु जन, नहीं आरम्भे काम, विपति देख छोड़े तुरंत मध्यम मन कर श्याम।
पुरुष सिंह संकल्प कर, सहते विपति अनेक, 'बना' न छोड़े ध्येय को, रघुबर राखे टेक।।

रचितः मानव धर्म प्रणेता
सद्गुरु श्री रणछोड़दासजी महाराज

STUDY PACKAGE

Subject : Mathematics

Topic : The Point & Straight Lines

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5. Que. from Compt. Exams
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The Point & Straight Line

1. Distance Formula:

The distance between the points $A(x_1,y_1)$ and $B(x_2,y_2)$ is $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$.

Solved Example # 1 **Find the value of x, if the distance between the points (x, –1) and (3, 2) is 5**

Solution.

Let P(x ,–1) and Q(3, 2) be the given points. Then PQ = 5 (given)

$\sqrt{(x-3)^2+(-1-2)^2} = 5 \quad \Rightarrow \quad (x-3)^2 + 9 = 25 \quad \Rightarrow \quad$ **x = 7 or x = – 1 Ans.**

Self practice problems :

1. Show that four points (0, –1), (6, 7) (–2, 3) and (8, 3) are the vertices of a rectangle.
2. Find the coordinates of the circumcenter of the triangle whose vertices are (8, 6), (8, –2) and (2, –2). Also find its circumradius. **Ans.** (5, 2), 5

2. Section Formula :

If P(x, y) divides the line joining $A(x_1, y_1)$ & $B(x_2, y_2)$ in the ratio m : n, then;

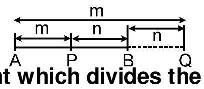
$$x = \frac{mx_2+nx_1}{m+n} \; ; \; y = \frac{my_2+ny_1}{m+n} .$$

NOTE : (i) If $\frac{m}{n}$ is positive, the division is internal, but if $\frac{m}{n}$ is negative, the division is external.

(ii) If P divides AB internally in the ratio m : n & Q divides AB externally in the ratio m : n then P & Q are said to be **harmonic conjugate** of each other w.r.t. AB.

Mathematically,

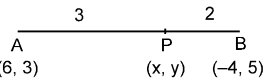
$\frac{2}{AB} = \frac{1}{AP} + \frac{1}{AQ}$ i.e. AP, AB & AQ are in H.P.



Solved Example# 2 **Find the coordinates of the point which divides the line segment joining the points (6, 3) and (–4, 5) in the ratio 3 : 2 (i) internally and (ii) externally.**

Solution. Let P (x, y) be the required point.

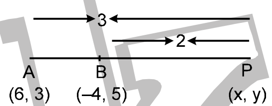
(i) For internal division :



$$x = \frac{3\times -4+2\times 6}{3+2} \text{ and } y = \frac{3\times 5+2\times 3}{3+2} \text{ or } x = 0 \text{ and } y = \frac{21}{5}$$

So the coordinates of P are $\left(\mathbf{0}, \frac{\mathbf{21}}{\mathbf{5}}\right)$ **Ans.**

(ii) For external division



$$x = \frac{3\times -4-2\times 6}{3-2} \text{ and } y = \frac{3\times 5-2\times 3}{3-2}$$

or x = –24 and y = 9

So the coordinates of P are **(–24, 9)** **Ans.**

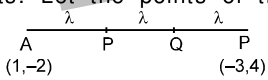
Solved Example # 3

Find the coordinates of points which trisect the line segment joining (1, – 2) and (– 3, 4).

Solution.

Let A (1, –2) and B(–3, 4) be the given points. Let the points of trisection be P and Q. Then

AP = PQ = QB = λ (say)



∴ PB = PQ + QB = 2λ and AQ = AP + PQ = 2λ

⇒ AP : PB = λ : 2λ = 1 : 2 and AQ : QB = 2λ : λ = 2 : 1

So P divides AB internally in the ratio 1 : 2 while Q divides internally in the ratio 2 : 1

∴ the coordinates of P are $\left(\frac{1\times -3+2\times 1}{1+2}, \frac{1\times 4+2\times -2}{1+2}\right)$ or $\left(-\frac{1}{3}, 0\right)$

and the coordinates of Q are $\left(\frac{2\times -3+1\times 1}{2+1}, \frac{2\times 4+1\times(-2)}{2+1}\right)$ or $\left(-\frac{5}{3}, 2\right)$

Hence, the points of trisection are $\left(-\frac{\mathbf{1}}{\mathbf{3}}, \mathbf{0}\right)$ and $\left(-\frac{\mathbf{5}}{\mathbf{3}}, \mathbf{2}\right)$ **Ans.**

Self practice problems :

3. In what ratio does the point (–1, –1) divide the line segment joining the points (4, 4) and (7, 7)? **Ans.** 5 : 8 externally
4. The three vertices of a parallelogram taken in order are (–1, 0), (3, 1) and (2, 2) respectively. Find the coordinates of the fourth vertex. **Ans.** (–2, 1)

3. Centroid, Incentre & Excentre:

If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of triangle ABC, whose sides BC, CA, AB are of lengths a, b, c respectively, then the co-ordinates of the special points of triangle ABC are as follows :

Centroid $G \equiv \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

Incentre $I \equiv \left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$,and Excentre (to A) $I_1 \equiv \left(\frac{-ax_1+bx_2+cx_3}{-a+b+c}, \frac{-ay_1+by_2+cy_3}{-a+b+c}\right)$ and so on.

NOTE :

(i) Incentre divides the angle bisectors in the ratio, (b + c) : a; (c + a) : b & (a + b) : c.

(ii) Incentre and excentre are harmonic conjugate of each other w.r.t. the angle bisector on which they lie.

(iii) Orthocenter, Centroid & Circumcenter are always collinear & centroid divides the line joining orthocentre & circumcenter in the ratio 2 : 1.

(iv) In an isosceles triangle G, O, I & C lie on the same line and in an equilateral triangle, all these four points coincide.

Sol. Ex. 4 Find the coordinates of (i) centroid (ii) in-centre of the triangle whose vertices are (0, 6), (8, 12) and (8, 0).

Solution (i) We know that the coordinates of the centroid of a triangle whose angular points are (x_1, y_1), (x_2, y_2) (x_3, y_3) are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

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So the coordinates of the centroid of a triangle whose vertices are (0, 6), (8, 12) and (8, 0) are $\left(\frac{0+8+8}{3}, \frac{6+12+0}{3}\right)$ or

$\left(\mathbf{\frac{16}{3}, 6}\right)$ **Ans.**

(ii) Let A (0, 6), B (8, 12) and C(8,) be the vertices of triangle ABC.

Then $c = AB = \sqrt{(0-8)^2+(6-12)^2} = 10$, $b = CA = \sqrt{(0-8)^2+(6-0)^2} = 10$

and $a = BC = \sqrt{(8-8)^2+(12-0)^2} = 12$.

The coordinates of the in-centre are $\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$

or $\left(\frac{12\times0+10\times8+10\times8}{12+10+10}, \frac{12\times6+10\times12+10\times0}{12+10+10}\right)$

or $\left(\frac{160}{32}, \frac{192}{32}\right)$ or **(5, 6)** **Ans.**

Self practice problems :

5. Two vertices of a triangle are (3, –5) and (–7, 4). If the centroid is (2, –1), find the third vertex. **Ans.**(10, – 2)

6. Find the coordinates of the centre of the circle inscribed in a triangle whose vertices are (– 36, 7), (20, 7) and (0, – 8) **Ans.** (–1, 0)

4. Area of a Triangle:

If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of triangle ABC, then its area is equal to

$\Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$, provided the vertices are considered in the counter clockwise sense. The above formula will give a (–) ve area if the vertices (x_i, y_i), i = 1, 2, 3 are placed in the clockwise sense.

NOTE : Area of n-sided polygon formed by points (x_1, y_1) ; (x_2, y_2); ....(x_n, y_n) is given by

$$\frac{1}{2}\left(\begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & x_3 \\ y_2 & y_3 \end{vmatrix} + \ldots\ldots\ldots\ldots + \begin{vmatrix} x_{n-1} & x_n \\ y_{n-1} & y_n \end{vmatrix} + \begin{vmatrix} x_n & x_1 \\ y_n & y_1 \end{vmatrix}\right)$$

Solved Example # 5: If the coordinates of two points A and B are (3, 4) and (5, –2) respectively. Find the coordinates of any point P if PA = PB and Area of ΔPAB = 10.

Solution

Let the coordinates of P be (x, y). Then

PA = PB $\Rightarrow$ $PA^2 = PB^2$ $\Rightarrow$ $(x-3)^2 + (y-4)^2 = (x-5)^2 + (y+2)^2$

$\Rightarrow$ $x - 3y - 1 = 0$

Now, Area of ΔPAB = 10 $\Rightarrow$ $\frac{1}{2}\begin{vmatrix} x & y & 1 \\ 3 & 4 & 1 \\ 5 & -2 & 1 \end{vmatrix} = \pm 10 \Rightarrow 6x + 2y - 26 = \pm 20$

$\Rightarrow$ $6x + 2y - 46 = 0$ or $6x + 2y - 6 = 0$

$\Rightarrow$ $3x + y - 23 = 0$ or $3x + y - 3 = 0$

Solving $x - 3y - 1 = 0$ and $3x + y - 23 = 0$ we get x = 7, y = 2. Solving $x - 3y - 1 = 0$ and $3x + y - 3 = 0$, we get x = 1, y = 0. Thus, the coordinates of P are **(7, 2) or (1, 0)** **Ans.**

Self practice problems :

7. The area of a triangle is 5. Two of its vertices are (2, 1) and (3, –2). The third vertex lies on y = x + 3. Find the third vertex. **Ans.** $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(-\frac{3}{2}, \frac{3}{2}\right)$

8. The vertices of a quadrilateral are (6, 3), (–3, 5), (4, –2) and (x, 3x) and are denoted by A, B, C and D, respectively. Find the values of x so that the area of triangle ABC is double the area of triangle DBC.

Ans. $x = \frac{11}{8}$ or $-\frac{3}{8}$

5. Slope Formula:

If θ is the angle at which a straight line is inclined to the positive direction of x–axis, & $0° \le \theta < 180°$, $\theta \ne 90°$, then the slope of the line, denoted by m, is defined by $m = \tan\theta$. If θ is 90°, m does not exist, but the line is parallel to the y–axis. If θ = 0, then m = 0 & the line is parallel to the x-axis.

If A (x_1, y_1) & B (x_2, y_2), $x_1 \ne x_2$, are points on a straight line, then the slope m of the line is given by : $m = \left(\frac{y_1 - y_2}{x_1 - x_2}\right)$.

Solved Example # 6: What is the slope of a line whose inclination is :
(i) 0º (ii) 90º (iii) 120º (iv) 150º

Solution (i) Here $\theta = 0º$

Slope = $\tan\theta = \tan 0º =$ **0** **Ans.**

(ii) Here $\theta = 90º$

$\therefore$ The slope of line is **not defined** **Ans.**

(iii) Here $\theta = 120º$

$\therefore$ Slope = $\tan\theta = \tan 120º = \tan(180º - 60º) = -\tan 30º =$ $\mathbf{-\sqrt{3}}$ **Ans.**

(iv) Here $\theta = 150º$

$\therefore$ Slope = $\tan\theta = \tan 150º = \tan(180º - 30º) = -\tan 30º = -\mathbf{\frac{1}{\sqrt{3}}}$ **Ans.**

Solved Example # 7 : Find the slope of the line passing through the points :
(i) (1, 6) and (– 4, 2) (ii) (5, 9) and (2, 9)

Solution

(i) Let A = (1, 6) and B = (– 4, – 2)

$\therefore$ Slope of AB = $\frac{2-6}{-4-1} = \frac{-4}{-5} = \mathbf{\frac{4}{5}}$ **Ans.** $\left(\text{Using slope} = \frac{y_2 - y_1}{x_2 - x_1}\right)$

(ii) Let A = (5, 9), B = (2, 9)

$\therefore$ Slope of AB = $\frac{9-9}{2-5} = \frac{0}{-3} =$ **0** **Ans.**

Self practice problems :

9. Find the value of x, if the slope of the line joining (1, 5) and (x, –7) is 4. **Ans.** – 2

10. What is the inclination of a line whose slope is

(i) 0 (ii) 1 (iii) –1 (iv) $-1/\sqrt{3}$

Ans. (i) 0º, (ii) 45º, (iii) 135º, (iv) 150º

6. Condition of collinearity of three points:

Points A (x_1, y_1), B (x_2, y_2), C(x_3, y_3) are collinear if

(i) $m_{AB} = m_{BC} = m_{CA}$ i.e. $\left(\dfrac{y_1 - y_2}{x_1 - x_2}\right) = \left(\dfrac{y_2 - y_3}{x_2 - x_3}\right)$ (ii) Δ ABC = 0 i.e. $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

(iii) AC = AB + BC or AB ~ BC (iv) A divides the line segment BC in some ratio.

Solved Example # 8 Show that the points (1, 1), (2, 3) and (3, 5) are collinear.

Solution.

Let (1, 1) (2, 3) and (3, 5) be the coordinates of the points A, B and C respectively.

Slope of AB = $\dfrac{3-1}{2-1}$ = 2 and Slope of BC = $\dfrac{5-3}{3-2}$ = 2

$\therefore$ Slope of AB = slope of AC

$\therefore$ AB & BC are parallel $\therefore$ A, B, C are collinear because B is on both lines AB and BC.

Self practice problem :

11. Prove that the points (a, 0), (0, b) and (1, 1) are collinear if $\dfrac{1}{a} + \dfrac{1}{b} = 1$

7. Equation of a Straight Line in various forms:

(i) **Point - Slope form :** $y - y_1 = m\,(x - x_1)$ is the equation of a straight line whose slope is m & which passes through the point (x_1, y_1).

Solved Example # 9 : Find the equation of a line passing through (2, –3) and inclined at an angle of 135º with the positive direction of x-axis.

Solution.

Here, m = slope of the line = tan 135º = tan (90º + 45º) = – cot 45º = –1, $x_1 = 2$, $y_1 = -3$

So, the equation of the line is $y - y_1 = m\,(x - x_1)$

i.e. y – (–3) = –1 (x – 2) or y + 3 = –x + 2 or **x + y + 1 = 0** **Ans.**

Self practice problem :

12. Find the equation of the perpendicular bisector of the line segment joining the points A(2, 3) and B (6, –5). **Ans.** x – 2 y – 6 = 0

(ii) **Slope – intercept form :** y = mx + c is the equation of a straight line whose slope is m & which makes an intercept c on the y–axis.

Solved Example # 10: Find the equation of a line with slope –1 and cutting off an intercept of 4 units on negative direction of y-axis.

Solution. Here m = –1 and c = – 4. So, the equation of the line is y = mx + c i.e. y = – x – 4 or **x + y + 4 = 0** **Ans.**

Self practice problem :

13. Find the equation of a straight line which cuts off an intercept of length 3 on y-axis and is parallel to the line joining the points (3, –2) and (1, 4). **Ans.** 3x + y – 3 = 0

(iii) **Two point form :** $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ is the equation of a straight line which passes through the points (x_1, y_1) & (x_2, y_2).

Solved Example # 11 Find the equation of the line joining the points (– 1, 3) and (4, – 2)

Solution.

Here the two points are $(x_1, y_1) = (-1, 3)$ and $(x_2, y_2) = (4, -2)$.

So, the equation of the line in two-point form is

$y - 3 = \dfrac{3-(-2)}{-1-4}(x+1) \Rightarrow y - 3 = -x - 1 \Rightarrow$ **x + y – 2 = 0** **Ans.**

Self practice problem :

14. Find the equations of the sides of the triangle whose vertices are (–1, 8), (4, –2) and (–5, –3). Also find the equation of the median through (–1, 8)

Ans. 2x + y – 6 = 0, x – 9y – 22 = 0, 11x – 4y + 43 = 0, 21x + y + 13 = 0

(iv) **Determinant form :** Equation of line passing through (x_1, y_1) and (x_2, y_2) is $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

Solved Example # 12

Find the equation of line passing through (2, 4) & (– 1, 3).

Solution.

$\begin{vmatrix} x & y & 1 \\ 2 & 4 & 1 \\ -1 & 3 & 1 \end{vmatrix} = 0 \Rightarrow$ **x – 3y + 10 = 0** **Ans.**

Self practice problem :

15. Find the equation of the passing through (– 2, 3) & (– 1, – 1).

Ans. 4x + y + 5 = 0

(v) **Intercept form :** $\dfrac{x}{a} + \dfrac{y}{b} = 1$ is the equation of a straight line which makes intercepts a & b on OX & OY respectively.

Solved Example # 13: Find the equation of the line which passes through the point (3, 4) and the sum of its intercepts on the axes is 14.

Sol. Let the equation of the line be $\dfrac{x}{a} + \dfrac{y}{b} = 1$ ....(i)

This passes through (3, 4), therefore $\dfrac{3}{a} + \dfrac{4}{b} = 1$ ....(ii)

It is given that a + b = 14 $\Rightarrow$ b = 14 – a. Putting b = 14 – a in (ii), we get $\dfrac{3}{a} + \dfrac{4}{14-a} = 1$

$\Rightarrow a^2 - 13a + 42 = 0$

$\Rightarrow$ (a – 7) (a – 6) = 0 $\Rightarrow$ a = 7, 6

For a = 7, b = 14 – 7 = 7 and for a = 6, b = 14 – 6 = 8.

Putting the values of a and b in (i), we get the equations of the lines

$\dfrac{x}{7} + \dfrac{y}{7} = 1$ and $\dfrac{y}{6} + \dfrac{y}{8} = 1$ or **x + y = 7 and 4x + 3y = 24** **Ans.**

Self practice problem :

16. Find the equation of the line through (2, 3) so that the segment of the line intercepted between the axes is bisected at this point. **Ans.** $3x + 2y = 12$.

(vi) **Perpendicular/Normal form :** $x\cos\alpha + y\sin\alpha = p$ (where $p > 0$, $0 \le \alpha < 2\pi$) is the equation of the straight line where the length of the perpendicular from the origin O on the line is p and this perpendicular makes an angle α with positive x–axis.

Solved Example # 14: Find the equation of the line which is at a distance 3 from the origin and the perpendicular from the origin to the line makes an angle of 30º with the positive direction of the x-axis.

Solution. Here $p = 3$, $\alpha = 30º$

$\therefore$ Equation of the line in the normal form is

$x\cos 30º + y\sin 30º = 3$ or $x\dfrac{\sqrt{3}}{2} + \dfrac{y}{2} = 3$ or $\mathbf{\sqrt{3}\,x + y = 6}$ **Ans.**

Self practice problem :

17. The length of the perpendicular from the origin to a line is 7 and the line makes an angle of 150º with the positive direction of y-axis. Find the equation of the line. **Ans.** $\sqrt{3}\,x + y - 14 = 0$

(vii) **Parametric form :** $P(r) = (x, y) = (x_1 + r\cos\theta,\ y_1 + r\sin\theta)$ or $\dfrac{x - x_1}{\cos\theta} = \dfrac{y - y_1}{\sin\theta} = r$ is the equation of the line in parametric form, where 'r' is the parameter whose absolute value is the distance of any point (x, y) on the line from the fixed point (x_1, y_1) on the line.

Solved Example # 15: Find the equation of the line through the point A(2, 3) and making an angle of 45º with the x-axis. Also determine the length of intercept on it between A and the line x + y + 1 = 0

Solution. The equation of a line through A and making an angle of 45º with the x-axis is

$$\frac{x-2}{\cos 45º} = \frac{y-3}{\sin 45º} \text{ or } \frac{x-2}{\frac{1}{\sqrt{2}}} = \frac{y-3}{\frac{1}{\sqrt{2}}} \quad \text{or} \quad x - y + 1 = 0$$

Suppose this line meets the line $x + y + 1 = 0$ at P such that AP = r. Then the coordinates of P are given by

$$\frac{x-2}{\cos 45º} = \frac{y-3}{\sin 45º} = r \Rightarrow \quad x = 2 + r\cos 45º,\ y = 3 + r\sin 45º$$

$$\Rightarrow \quad x = 2 + \frac{r}{\sqrt{2}},\ y = 3 + \frac{r}{\sqrt{2}}$$

Thus, the coordinates of P are $\left(2 + \dfrac{r}{\sqrt{2}}, 3 + \dfrac{r}{\sqrt{2}}\right)$

Since P lies on $x + y + 1 = 0$, so $2 + \dfrac{r}{\sqrt{2}} + 3 + \dfrac{r}{\sqrt{2}} + 1 = 0$

$\Rightarrow \quad \sqrt{2}\,r = -6 \Rightarrow r = -3\sqrt{2} \Rightarrow$ length AP = | r | = $3\sqrt{2}$

Thus, the length of the intercept = $\mathbf{3\sqrt{2}}$ **Ans.**

Self practice problem :

18. A straight line is drawn through the point $A(\sqrt{3}, 2)$ making an angle of $\pi/6$ with positive direction of the x-axis. If it meets the straight line $\sqrt{3x} - 4y + 8 = 0$ in B, find the distance between A and B. **Ans.** 6 units

(viii) **General Form :** $ax + by + c = 0$ is the equation of a straight line in the general form

In this case, slope of line $= -\dfrac{a}{b}$

x - intercept $= -\dfrac{c}{a}$

y - intercept $= -\dfrac{c}{b}$

Solved Example # 16 Find slope, x-intercept & y-intercept of the line 2x – 3y + 5 = 0.

Solution. Here, $a = 2$, $b = -3$, $c = 5$

$\therefore$ slope $= -\dfrac{a}{b} = \mathbf{\dfrac{2}{3}}$ **Ans.**

x-intercept $= -\dfrac{c}{a} = -\mathbf{\dfrac{5}{2}}$ **Ans.**

y-intercept $= \mathbf{\dfrac{5}{3}}$ **Ans.**

Self practice problem :

19. Find the slope, x-intercept & y-intercept of the line $3x - 5y - 8 = 0$. **Ans** $\dfrac{3}{5}, \dfrac{8}{3}, -\dfrac{8}{5}$

8. Angle between two straight lines in terms of their slopes:

If m_1 & m_2 are the slopes of two intersecting straight lines ($m_1 m_2 \ne -1$) & θ is the acute angle between them, then $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$.

NOTE :

(i) Let m_1, m_2, m_3 are the slopes of three lines $L_1 = 0; L_2 = 0; L_3 = 0$ where $m_1 > m_2 > m_3$ then the interior angles of the Δ ABC found by these lines are given by,

$$\tan A = \frac{m_1 - m_2}{1 + m_1 m_2}\ ;\ \tan B = \frac{m_2 - m_3}{1 + m_2 m_3} \ \& \ \tan C = \frac{m_3 - m_1}{1 + m_3 m_1}$$

(ii) The equation of lines passing through point (x_1, y_1) and making angle α with the line $y = mx + c$ are given by :

$(y - y_1) = \tan(\theta - \alpha)(x - x_1)$ &

$(y - y_1) = \tan(\theta + \alpha)(x - x1)$, where $\tan\theta = m$.

Solved Example # 17: The acute angle between two lines is $\pi/4$ and slope of one of them is 1/2. Find the slope of the other line.

Solution.

If θ be the acute angle between the lines with slopes m_1 and m_2, then $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

Let $\theta = \frac{\pi}{4}$ and $m_1 = \frac{1}{2}$

$\therefore$ $\tan\frac{\pi}{4} = \left|\frac{\frac{1}{2} - m_2}{1+\frac{1}{2}m_2}\right|$ $\Rightarrow$ $1 = \left|\frac{1-2m_2}{2+m_2}\right|$ $\Rightarrow$ $\frac{1-2m_2}{2+m_2} = +1$ or -1

Now $\frac{1-2m_2}{2+m_2} = 1$ $\Rightarrow m_2 = -\frac{1}{3}$ and $\frac{1-2m_2}{2+m_2} = -1$ $\Rightarrow$ $m_2 = 3$.

$\therefore$ The slope of the other line is either **– 1/3 or 3** **Ans.**

Solved Example # 18: Find the equation of the straight line which passes through the origin and making angle 60º with the line $x + \sqrt{3}\,y + 3\sqrt{3} = 0$.

Solution. Given line is $x + \sqrt{3}\,y + 3\sqrt{3} = 0$.

$\Rightarrow$ $y = \left(-\frac{1}{\sqrt{3}}\right)x - 3$ $\therefore$ Slope of (1) $= -\frac{1}{\sqrt{3}}$.

Let slope of the required line be m. Also between these lines is given to be 60º.

$\Rightarrow$ $\tan 60º = \left|\frac{m-(-1/\sqrt{3})}{1+m(-1/\sqrt{3})}\right|$ $\Rightarrow$ $\sqrt{3} = \left|\frac{\sqrt{3}m+1}{\sqrt{3}-m}\right|$ $\Rightarrow$ $\frac{\sqrt{3}m+1}{\sqrt{3}-m} = \pm\sqrt{3}$

$\frac{\sqrt{3}m+1}{\sqrt{3}-m} = -\sqrt{3}$ $\Rightarrow$ $\sqrt{3}\,m + 1 = 3 - \sqrt{3}\,m \Rightarrow$ $m = \frac{1}{\sqrt{3}}$

Using y = mx + c, the equation of the required line is

$y = \frac{1}{\sqrt{3}}x + 0$ i.e. $x - \sqrt{3}\,y = 0$. ($\because$ This passes through origin, so c = 0)

$\frac{\sqrt{3}m+1}{\sqrt{3}-m} = -\sqrt{3}$ $\Rightarrow$ $\sqrt{3}\,m + 1 = -3 + \sqrt{3}\,m$

$\Rightarrow$ m is not defined

$\therefore$ The slope of the required line is not defined. Thus, the required line is a vertical line. This line is to pass through the origin.

$\therefore$ The equation of the required line is **x = 0 Ans.**

Self practice problem :

20. A vertex of an equilateral triangle is (2, 3) and the equation of the opposite side is x + y = 2. Find the equation of the other sides of the triangle. **Ans.** $(2 + \sqrt{3})x - y + 2\sqrt{3} - 1 = 0$ and $(2 + \sqrt{3})\,x - y - 2\sqrt{3} - 1 = 0$.

9. Parallel Lines:

(i) When two straight lines are parallel their slopes are equal. Thus any line parallel to y = mx + c is of the type y = mx + d, where k is a parameter.

(ii) Two lines ax + by + c = 0 and $a'x + b'y + c' = 0$ are parallel if $\frac{a}{a'} = \frac{b}{b'} \neq \frac{c}{c'}$.

Thus any line parallel to ax + by + c = 0 is of the type ax + by + k = 0, where k is a parameter.

(iii) The distance between two parallel lines with equations $ax + by + c_1 = 0$ & $ax + by + c_2 = 0$ is $\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$.

NOTE: Coefficients of x & y in both the equations must be same.

(iv) The area of the parallelogram $= \frac{p_1p_2}{\sin\theta}$, where p_1 & p_2 are distances between two pairs of opposite sides & θ is the angle between any two adjacent sides. Note that area of the parallelogram bounded by the lines $y = m_1x + c_1$, $y = m_1x + c_2$ and $y = m_2x + d_1$, $y = m_2x + d_2$ is given by $\left|\frac{(c_1-c_2)(d_1-d_2)}{m_1-m_2}\right|$.

Sol. Ex. 19: Find the equation of the straight line that has y-intercept 4 and is parallel to the straight line 2x – 3y = 7.

Solution. Given line is 2x – 3y = 7

(1) $\Rightarrow$ $3y = 2x - 7 \Rightarrow$ $y = \frac{2}{3}x - \frac{7}{3}$ $\therefore$ Slope of (1) is 2/3

The required line is parallel to (1), so its slope is also 2/3, y-intercept of required line = 4

$\therefore$ By using y = mx + c form, the equation of the required line is

$y = \frac{2}{3}x + 4$ or **2x – 3y + 12 = 0 Ans.**

Solved Example # 20: Two sides of a square lie on the lines x + y = 1 and x + y + 2 = 0. What is its area?

Solution. Clearly the length of the side of the square is equal to the distance between the parallel lines

x + y – 1 = 0 ....(i) and x + y + 2 = 0 ....(ii)

Putting x = 0 in (i), we get y = 1. So (0, 1) is a point on line (i).

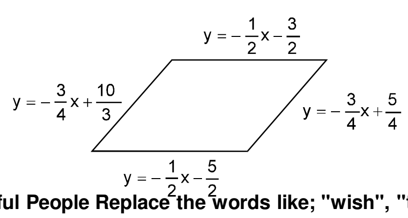
Now, Distance between the parallel lines

= length of the $\perp$ from (0, 1) to x + y + 2 = 0 $= \frac{|0+1+2|}{\sqrt{1^2+1^2}} = \frac{3}{\sqrt{2}}$

Thus, the length of the side of the square is $\frac{3}{\sqrt{2}}$ and hence its area $= \left(\frac{3}{\sqrt{2}}\right)^2 = \mathbf{\frac{9}{2}}$

Solved Example # 21: Find the area of the parallelogram whose sides are x + 2y + 3 = 0, 3x + 4y – 5 = 0, 2x + 4y + 5 = 0 and 3x + 4y – 10 = 0

Solution.



Here, $c_1 = -\frac{3}{2}$, $c_2 = \frac{5}{2}$, $d_1 = \frac{10}{3}$, $d_2 = -\frac{5}{2}$, $m_1 = -\frac{1}{2}$, $m_2 = -\frac{3}{4}$

$\therefore$ Area = $\left|\dfrac{\left(-\frac{3}{2}+\frac{5}{2}\right)\left(\frac{10}{3}+\frac{5}{2}\right)}{\left(-\frac{1}{2}+\frac{3}{4}\right)}\right| = \mathbf{\frac{70}{3}}$ **sq. units Ans.**

Self practice problem :

21. Find the area of parallelogram whose sides are given by $4x - 5y + 1 = 0$, $x - 3y - 6 = 0$, $4x - 5y - 2 = 0$ and $2x - 6y + 5 = 0$ **Ans.** $\frac{51}{14}$ sq. units

10. Perpendicular Lines:

(i) When two lines of slopes m_1 & m_2 are at right angles, the product of their slopes is -1, i.e. $m_1 m_2 = -1$. Thus any line perpendicular to $y = mx + c$ is of the form $y = -\frac{1}{m}x + d$, where d is any parameter.

(ii) Two lines $ax + by + c = 0$ and $a'x + b'y + c' = 0$ are perpendicular if $aa' + bb' = 0$. Thus any line perpendicular to $ax + by + c = 0$ is of the form $bx - ay + k = 0$, where k is any parameter.

Solved Example # 22

Find the equation of the straight line that passes through the point (3, 4) and perpendicular to the line 3x + 2y + 5 = 0

Solution.

The equation of a line perpendicular to $3x + 2y + 5 = 0$ is

$2x - 3y + \lambda = 0$ ....(i)

This passes through the point (3, 4)

$\therefore$ $3 \times 2 - 3 \times 4 + \lambda = 0 \Rightarrow \lambda = 6$

Putting $\lambda = 6$ in (i), we get **$2x - 3y + 6 = 0$**, which is the required equation. **Ans.**

Aliter The slope of the given line is $-3/2$. Since the required line is perpendicular to the given line. So, the slope of the required line is $2/3$. As it passes through (3, 4). So, its equation is $y - 4 = \frac{2}{3}(x - 3)$ or **$2x - 3y + 6 = 0$ Ans.**

Self practice problem :

22. The vertices of a triangle are A(10, 4), B (–4, 9) and C(–2, –1). Find the equation of its altitudes. Also find its orthocentre.

Ans. $x - 5y + 10 = 0$, $12x + 5y + 3 = 0$, $14x - 5y + 23 = 0$, $\left(-1, \frac{9}{5}\right)$

11. Position of the point (x_1, y_1) relative of the line $ax + by + c = 0$:

If $ax_1 + by_1 + c$ is of the same sign as c, then the point (x_1, y_1) lie on the origin side of $ax + by + c = 0$. But if the sign of $ax_1 + by_1 + c$ is opposite to that of c, the point (x_1, y_1) will lie on the non–origin side of $ax + by + c = 0$.

In general two points (x_1, y_1) and (x_2, y_2) will lie on same side or opposite side of $ax + by + c = 0$ according as $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ are of same or opposite sign respectively.

Solved Example # 23

Show that (1, 4) and (0, –3) lie on the opposite sides of the line x + 3y + 7 = 0.

Solution.

At (1, 4), the value of $x + 3y + 7 = 1 + 3(4) + 7 = 20 > 0$.

At (0, – 3), the value of $x + 3y + 7 = 0 + 3(-3) + 7 = -2 < 0$

$\therefore$ **The points (1, 4) and (0, – 3) are on the opposite sides of the given line. Ans.**

Self practice problems :

23. Are the points (3, – 4) and (2, 6) on the same or opposite side of the line $3x - 4y = 8$?

Ans. Opposite sides

24. Which one of the points (1, 1), (–1, 2) and (2, 3) lies on the side of the line $4x + 3y - 5 = 0$ on which the origin lies?

Ans. (–1, 2)

12. The ratio in which a given line divides the line segment joining two points:

Let the given line $ax + by + c = 0$ divide the line segment joining $A(x_1, y_1)$ & $B(x_2, y_2)$ in the ratio m : n, then $\frac{m}{n} = -\frac{ax_1+by_1+c}{ax_2+by_2+c}$.

If A & B are on the same side of the given line then m/n is negative but if A & B are on opposite sides of the given line, then m/n is positive

Solved Example # 24

Find the ratio in which the line joining the points A (1, 2) and B(– 3, 4) is divided by the line x + y – 5 = 0.

Solution. Let the line $x + y = 5$ divides AB in the ratio k : 1 at P

$\therefore$ coordinate of P are

$\left(\frac{-3k+1}{k+1}, \frac{4k+2}{k+1}\right)$

Since P lies on $x + y - 5 = 0$

$\therefore$ $\frac{-3k+1}{k+1} + \frac{4k+2}{k+1} - 5 = 0.$ $\Rightarrow$ $k = -\frac{1}{2}$

$\therefore$ **Required ratio is 1 : 2 extrenally Ans.**

Aliter Let the ratio is m : n

$\therefore$ $\frac{m}{n} = -\frac{(1\times1+1\times2-5)}{1\times(-3)+1\times4-5} = -\frac{1}{2}$ $\therefore$ **ratio is 1 : 2 externally Ans.**

Self practice problem :

25. If the line $2x - 3y + \lambda = 0$ divides the line joining the points A (– 1, 2) & B(– 3, – 3) internally in the ratio 2 : 3, find λ.

Ans. $\frac{18}{5}$

13. Length of perpendicular from a point on a line:

The length of perpendicular from $P(x_1, y_1)$ on $ax + by + c = 0$ is $\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right|$.

Solved Example # 25 Find the distance between the line 12x – 5y + 9 = 0 and the point (2, 1)

Solution. The required distance = $\left|\frac{12\times2-5\times1+9}{\sqrt{12^2+(-5)^2}}\right| = \frac{|24-5+9|}{13} = \mathbf{\frac{28}{13}}$ **Ans.**

Solved Example # 26
Find all points on x + y = 4 that lie at a unit distance from the line 4x + 3y – 10 = 0.

Solution. Note that the coordinates of an arbitrary point on x + y = 4 can be obtained by putting x = t (or y = t) and then obtaining y (or x) from the equation of the line, where t is a parameter.
Putting x = t in the equation x + y = 4 of the given line, we obtain y = 4 – t. So, coordinates of an arbitrary point on the given line are P(t, 4 – t). Let P(t, 4 – t) be the required point. Then, distance of P from the line 4x + 3y – 10 = 0 is unity i.e.

$\Rightarrow \quad \left|\dfrac{4t+3(4-t)-10}{\sqrt{4^2+3^2}}\right| = 1 \Rightarrow |t+2| = 5 \Rightarrow t + 2 = \pm 5$

$\Rightarrow \quad t = -7$ or $t = 3$ Hence, required points are **(–7, 11) and (3, 1)** **Ans.**

Self practice problem :

26. Find the length of the altitudes from the vertices of the triangle with vertices :(–1, 1), (5, 2) and (3, –1).

Ans. $\dfrac{16}{\sqrt{13}}, \dfrac{8}{\sqrt{5}}, \dfrac{16}{\sqrt{37}}$

14. Reflection of a point about a line:

(i) Foot of the perpendicular from a point on the line is $\dfrac{x-x_1}{a} = \dfrac{y-y_1}{b} = -\dfrac{ax_1+by_1+c}{a^2+b^2}$

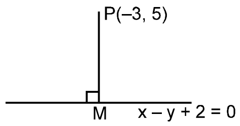
(ii) The image of a point (x_1, y_1) about the line ax + by + c = 0 is

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = -2\frac{ax_1+by_1+c}{a^2+b^2}.$$

Solved Example # 27 Find the foot of perpendicular of the line drawn from P (– 3, 5) on the line x – y + 2 = 0.

Solution.

Slope of PM = – 1



$\therefore$ Equation of PM is
x + y – 2 = 0 ....(i)
solving equation (i) with x – y + 2 = 0, we get coordinates of **M (0, 2)** **Ans.**

Aliter Here, $\dfrac{x+3}{1} = \dfrac{y-5}{-1} = -\dfrac{(1\times(-3)+(-1)\times 5+2)}{(1)^2+(-1)^2}$

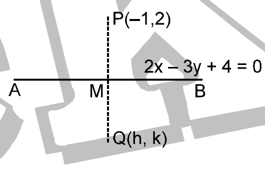
$\Rightarrow \quad \dfrac{x+3}{1} = \dfrac{y-5}{-1} = 3 \quad \Rightarrow \quad x + 3 = 3 \quad \Rightarrow \quad x = 0$

and $\quad y - 5 = -3 \quad \Rightarrow \quad y = 2$

$\therefore$ **M is (0, 2)** **Ans.**

Solved Example # 28
Find the image of the point P(–1, 2) in the line mirror 2x – 3y + 4 = 0.

Solution. Let image of P is Q.



$\therefore$ PM = MQ & PQ $\perp$ AB
Let Q is (h, k)

$\therefore$ M is $\left(\dfrac{h-1}{2}, \dfrac{k+2}{2}\right)$

It lies on 2x – 3y + 4 = 0.

$\therefore \quad 2\left(\dfrac{h-1}{2}\right) - 3\left(\dfrac{k+2}{2}\right) + 4 = 0.$

or 2h – 3k = 0 ....(i)

slope of PQ = $\dfrac{k-2}{h+1}$

PQ $\perp$ AB

$\therefore \quad \dfrac{k-2}{h+1} \times \dfrac{2}{3} = -1.$

$\Rightarrow$ 3h + 2k – 1 = 0. ....(ii)

soving (i) & (ii), we get

$h = \dfrac{3}{13}, k = \dfrac{2}{13}$

$\therefore$ Image of P(– 1, 2) is $Q\left(\mathbf{\dfrac{3}{13}, \dfrac{2}{13}}\right)$ **Ans.**

Aliter The image of P (– 1, 2) about the line 2x – 3y + 4 = 0 is

$\dfrac{x+1}{2} = \dfrac{y-2}{-3} = -2\dfrac{[2(-1)-3(2)+4]}{2^2+(-3)^2}$

$\dfrac{x+1}{2} = \dfrac{y-2}{-3} = \dfrac{8}{13}$

$\Rightarrow \quad 13x + 13 = 16 \quad \Rightarrow \quad x = \dfrac{3}{13}$

& $\quad 13y - 26 = -24 \quad \Rightarrow \quad y = \dfrac{2}{13} \quad \therefore$ image is $\left(\mathbf{\dfrac{3}{13}, \dfrac{2}{13}}\right)$ **Ans.**

Self practice problems :

27. Find the foot of perpendicular of the line drawn from (– 2, – 3) on the line 3x – 2y – 1 = 0. **Ans.** $\left(\dfrac{-23}{13}, \dfrac{-41}{13}\right)$

28. Find the image of the point (1, 2) in y-axis. **Ans.** (– 1, 2)

15. Bisectors of the angles between two lines:

Equations of the bisectors of angles between the lines $ax + by + c = 0$ &

$a'x + b'y + c' = 0$ $(ab' \neq a'b)$ are : $\dfrac{ax+by+c}{\sqrt{a^2+b^2}} = \pm \dfrac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$

NOTE :

Equation of straight lines passing through $P(x_1, y_1)$ & equally inclined with the lines $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$ are those which are parallel to the bisectors between these two lines & passing through the point P.

Solved Example # 29

Find the equations of the bisectors of the angle between the straight lines $3x - 4y + 7 = 0$ and $12x - 5y - 8 = 0$.

Solution.

The equations of the bisectors of the angles between $3x - 4y + 7 = 0$ and $12x - 5y - 8 = 0$ are $\dfrac{3x-4y+7}{\sqrt{3^2+(-4)^2}} = \pm \dfrac{12x-5y-8}{\sqrt{12^2+(-5)^2}}$

or $\dfrac{3x-4y+7}{5} = \pm \dfrac{12x-5y-8}{13}$

or $39x - 52y + 91 = \pm (60x - 25y - 8)$

Taking the positive sign, we get **$21x + 27y - 131 = 0$** as one bisector **Ans.**

Taking the negative sign, we get **$99x - 77y + 51 = 0$** as the other bisector. **Ans.**

Self practice problem :

29. Find the equations of the bisectors of the angles between the following pairs of straight lines $3x + 4y + 13 = 0$ and $12x - 5y + 32 = 0$

Ans. $21x - 77y - 9 = 0$ and $99x + 27y + 329 = 0$

16. Methods to discriminate between the acute angle bisector & the obtuse angle bisector:

(i) If θ be the angle between one of the lines & one of the bisectors, find $\tan\theta$.

If $|\tan\theta| < 1$, then $2\theta < 90°$ so that this bisector is the acute angle bisector.

If $|\tan\theta| > 1$, then we get the bisector to be the obtuse angle bisector.

(ii) Let $L_1 = 0$ & $L_2 = 0$ are the given lines & $u_1 = 0$ and $u_2 = 0$ are the bisectors between $L_1 = 0$ & $L_2 = 0$. Take a point P on any one of the lines $L_1 = 0$ or $L_2 = 0$ and drop perpendicular on $u_1 = 0$ & $u_2 = 0$ as shown. If,

$|p| < |q| \Rightarrow u_1$ is the acute angle bisector.

$|p| > |q| \Rightarrow u_1$ is the obtuse angle bisector.

$|p| = |q| \Rightarrow$ the lines L_1 & L_2 are perpendicular.

(iii) If $aa' + bb' < 0$, then the equation of the bisector of this acute angle is

$$\frac{ax+by+c}{\sqrt{a^2+b^2}} = + \frac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$$

If, however, $aa' + bb' > 0$, the equation of the bisector of the obtuse angle is :

$$\frac{ax+by+c}{\sqrt{a^2+b^2}} = + \frac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$$

Solved Example # 30

For the straight lines $4x + 3y - 6 = 0$ and $5x + 12y + 9 = 0$, find the equation of the

(i) bisector of the obtuse angle between them;

(ii) bisector of the acute angle between them;

Solution.

(i) The equations of the given straight lines are

$4x + 3y - 6 = 0$ ....(1)

$5x + 12y + 9 = 0$ ....(2)

The equation of the bisectors of the angles between lines (1) and (2) are

$\dfrac{4x+3y-6}{\sqrt{4^2+3^2}} = \pm \dfrac{5x+12y+9}{\sqrt{5^2+12^2}}$ or $\dfrac{4x+3y-6}{5} = \pm \dfrac{5x+12y+9}{13}$

Taking the positive sign, we have

$$\frac{4x+3y-6}{5} = \frac{5x+12y+9}{13}$$

or $52x + 39y - 78 = 25x + 60y + 45$ or $27x - 21y - 123 = 0$

or $9x - 7y - 41 = 0$

Taking the negative sign, we have

$$\frac{4x+3y-6}{5} = -\frac{5x+12y+9}{13}$$

or $52x + 39y - 78 = -25x - 60y - 45$ or $77x + 99y - 33 = 0$

or $7x + 9y - 3 = 0$

Hence the equation of the bisectors are

$9x - 7y - 41 = 0$ ....(3)

and $7x + 9y - 3 = 0$ ....(4)

Now slope of line (1) $= -\dfrac{4}{3}$ and slope of the bisector (3) $= \dfrac{9}{7}$.

If θ be the acute angle between the line (1) and the bisector (3), then

$$\tan\theta = \left|\frac{\frac{9}{7}+\frac{4}{3}}{1+\frac{9}{7}\left(-\frac{4}{3}\right)}\right| = \left|\frac{27+28}{21-36}\right| = \left|\frac{55}{-15}\right| = \frac{11}{3} > 1$$

$\therefore$ $\theta > 45º$

Hence **$9x - 7y - 41 = 0$** is the bisector of the obtuse angle between the given lines (1) and (2)

Ans.

(ii) Since $9x - 7y - 41$ is the bisector of the obtuse angle between the given lines, therefore the other bisector **$7x + 9y - 3 = 0$** will be the bisector of the acute angle between the given lines.

2nd Method :

Writing the equation of the lines so that constants become positive we have

$-4x - 3y + 6 = 0$ ....(1)

and $5x + 12y + 9 = 0$ ....(2)

Here $a_1 = -4$, $a_2 = 5$, $b_1 = -3$, $b_2 = 12$

Now $a_1a_2 + b_1b_2 = -20 - 36 = -56 < 0$

$\therefore$ origin does not lie in the obtuse angle between lines (1) and (2) and hence equation of the bisector of the obtuse angle between lines (1) and (2) will be

$$\frac{-4x-3y+6}{\sqrt{(-4)^2+(-3)^2}} = -\frac{5x+12y+9}{\sqrt{5^2+12^2}}$$

or $13(-4x - 3y + 6) = -5(5x + 12y + 9)$

or $27x - 21y - 123 = 0$ or $\mathbf{9x - 7y - 41 = 0}$ **Ans.**

and the equation of the bisector of the acute angle will be (origin lies in the acute angle)

$$\frac{-4x-3y+6}{\sqrt{(-4)^2+(-3)^2}} = \frac{5x+12y+9}{\sqrt{5^2+12^2}}$$

or $77x + 99y - 33 = 0$ or $\mathbf{7x + 9y - 3 = 0}$ **Ans.**

Self practice problem :

30. Find the equations of the bisectors of the angles between the lines $x + y - 3 = 0$ and $7x - y + 5 = 0$ and state which of them bisects the acute angle between the lines.

Ans. $x - 3y + 10 = 0$ (bisector of the obtuse angle); $4x + 1 = 0$ (bisector of the acute angle)

17. To discriminate between the bisector of the angle containing a point:

To discriminate between the bisector of the angle containing the origin & that of the angle not containing the origin. Rewrite the equations, $ax + by + c = 0$ & $a'x + b'y + c' = 0$ such that the constant terms c, c' are positive. Then ;

$\dfrac{ax+by+c}{\sqrt{a^2+b^2}} = +\dfrac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$ gives the equation of the bisector of the angle containing the origin & $\dfrac{ax+by+c}{\sqrt{a^2+b^2}} = -\dfrac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$ gives the equation of the bisector of the angle not containing the origin. In general equation of the bisector which contains the point (α, β) is

$$\frac{ax+by+c}{\sqrt{a^2+b^2}} = \frac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}} \text{ or } \frac{ax+by+c}{\sqrt{a^2+b^2}} = -\frac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$$ according as

$a\alpha + b\beta + c$ and $a'\alpha + b'\beta + c'$ having same sign or otherwise.

Solved Example # 31

For the straight lines $4x + 3y - 6 = 0$ and $5x + 12y + 9 = 0$, find the equation of the bisector of the angle which contains the origin.

Solution.

For point O(0, 0), $4x + 3y - 6 = -6 < 0$ and $5x + 12y + 9 = 9 > 0$

Hence for point O(0, 0) $4x + 3y - 6$ and $5x + 12y + 9$ are of opposite signs.

Hence equation of the bisector of the angle between the given lines containing the origin will be

$$\frac{4x+3y-6}{\sqrt{(4)^2+(3)^2}} = -\frac{5x+12y+9}{\sqrt{5^2+12^2}}$$

or $\dfrac{4x+3y-6}{5} = -\dfrac{5x+12y+9}{13}$

or $52x + 39y - 78 = -25x - 60y - 45$.

or $77x + 99y - 33 = 0$

or $\mathbf{7x + 9y - 3 = 0}$ **Ans.**

Self practice problem :

31. Find the equation of the bisector of the angle between the lines $x + 2y - 11 = 0$ and $3x - 6y - 5 = 0$ which contains the point $(1, -3)$. **Ans.** $3x - 19 = 0$

18. Condition of Concurrency:

Three lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ & $a_3x + b_3y + c_3 = 0$ are concurrent if

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0.$$

Alternatively : If three constants A, B & C (not all zero) can be found such that $A(a_1x + b_1y + c_1) + B(a_2x + b_2y + c_2) + C(a_3x + b_3y + c_3) \equiv 0$, then the three straight lines are concurrent.

Solved Example # 32

Prove that the straight lines $4x + 7y = 9$, $5x - 8y + 15 = 0$ and $9x - y + 6 = 0$ are concurrent.

Solution.

Given lines are

$4x + 7y - 9 = 0$ ....(1)

$5x - 8y + 15 = 0$ ....(2)

and $9x - y + 6 = 0$ ....(3)

$$\Delta = \begin{vmatrix} 4 & 7 & -9 \\ 5 & -8 & 15 \\ 9 & -1 & 6 \end{vmatrix} = 4(-48 + 15) - 7(30 - 135) - 9(-5 + 72) = -132 + 735 - 603 = 0$$

Hence lines (1), (2) and (3) are concurrent. **Proved**

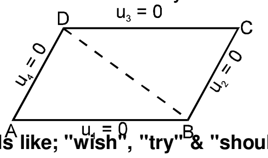
Self practice problem :

32. Find the value of m so that the lines $3x + y + 2 = 0$, $2x - y + 3 = 0$ and $x + my - 3 = 0$ may be concurrent.

Ans. 4

19. Family Of Straight Lines:

The equation of a family of straight lines passing through the point of intersection of the lines, $L_1 \equiv a_1x + b_1y + c_1 = 0$ & $L_2 \equiv a_2x + b_2y + c_2 = 0$ is given by $L_1 + k L_2 = 0$ i.e. $(a_1x + b_1y + c_1) + k(a_2x + b_2y + c_2) = 0$, where k is an arbitrary real number.



NOTE :

(i) If $u_1 = ax + by + c$, $u_2 = a'x + b'y + d$, $u_3 = ax + by + c'$, $u_4 = a'x + b'y + d'$ then $u_1 = 0; u_2 = 0; u_3 = 0; u_4 = 0$ form a parallelogram.
The diagonal BD can be given by $u_2u_3 - u_1u_4 = 0$.

(ii) The diagonal AC is also given by $u_1 + \lambda u_4 = 0$ and $u_2 + \mu u_3 = 0$, if the two equations are identical for some real λ and μ.
[For getting the values of λ & μ compare the coefficients of x, y & the constant terms].

Solved Example # 33
Find the equation of the straight line which passes through the point (2, –3) and the point of intersection of the lines x + y + 4 = 0 and 3x – y – 8 = 0.

Solution. Any line through the intersection of the lines $x + y + 4 = 0$ and $3x - y - 8 = 0$ has the equation
$(x + y + 4) + \lambda (3x - y - 8) = 0$ ....(i)
This will pass through (2, –3) if
$(2 - 3 + 4) + \lambda (6 + 3 - 8) = 0$ or $3 + \lambda = 0 \Rightarrow \lambda = -3$.
Putting the value of λ in (i), the required line is
$(x + y + 4) + (-3)(3x - y - 8) = 0$
or $-8x + 4y + 28 = 0$ or $\mathbf{2x - y - 7 = 0}$ **Ans.**

Aliter Solving the equations $x + y + 4 = 0$ and $3x - y - 8 = 0$ by cross-multiplication, we get $x = 1$, $y = -5$
So the two lines intersect at the point (1, –5). Hence the required line passes through (2, –3) and (1, –5) and so its equation is

$$y + 3 = -\frac{5+3}{1-2}(x - 2) \text{ or } \mathbf{2x - y - 7 = 0} \textbf{ Ans.}$$

Solved Example # 34
Obtain the equations of the lines passing through the intersection of lines 4x – 3y – 1 = 0 and 2x – 5y + 3 = 0 and equally inclined to the axes.

Solution. The equation of any line through the intersection of the given lines is
$(4x - 3y - 1) + \lambda (2x - 5y + 3) = 0$
or $x(2\lambda + 4) - y(5\lambda + 3) + 3\lambda - 1 = 0$ ....(i)

Let m be the slope of this line. Then $m = \dfrac{2\lambda+4}{5\lambda+3}$

As the line is equally inclined with the axes, therefore

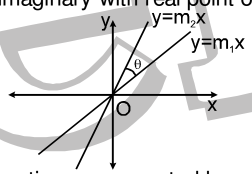
$m = \tan 45^\circ$ of $m = \tan 135^\circ \Rightarrow m = \pm 1$, $\dfrac{2\lambda+4}{5\lambda+3} = \pm 1 \Rightarrow \lambda = -1$ or $\dfrac{1}{3}$, putting the values of λ in (i), we get $2x + 2y - 4 = 0$ and $14x - 14y = 0$
i.e. x + y – 2 = 0 and x = y as the equations of the required lines. Ans.

Self practice problem :

33. Find the equation of the lines through the point of intersection of the lines $x - 3y + 1 = 0$ and $2x + 5y - 9 = 0$ and whose distance from the origin is $\sqrt{5}$ **Ans.** $2x + y - 5 = 0$

20. A Pair of straight lines through origin:

(i) A homogeneous equation of degree two,
"$ax^2 + 2hxy + by^2 = 0$" always represents a pair of straight lines passing through the origin if :
(a) $h^2 > ab \Rightarrow$ lines are real & distinct .
(b) $h^2 = ab \Rightarrow$ lines are coincident .
(c) $h^2 < ab \Rightarrow$ lines are imaginary with real point of intersection i.e. (0, 0)



(ii) If $y = m_1x$ & $y = m_2x$ be the two equations represented by $ax^2 + 2hxy + by^2 = 0$, then;
$m_1 + m_2 = -\dfrac{2h}{b}$ & $m_1 m_2 = \dfrac{a}{b}$.

(iii) If θ is the acute angle between the pair of straight lines represented by,
$ax^2 + 2hxy + by^2 = 0$, then ; $\tan\theta = \left|\dfrac{2\sqrt{h^2 - ab}}{a+b}\right|$.

(iv) The condition that these lines are :
(a) At right angles to each other is $a + b = 0$. i.e. co–efficient of x^2 + co–efficient of $y^2 = 0$.
(b) Coincident is $h^2 = ab$.
(c) Equally inclined to the axis of x is $h = 0$.i.e. coeff. of xy = 0 .

NOTE : A homogeneous equation of degree n represents n straight lines passing through origin.

(v) The equation to the pair of straight lines bisecting the angle between the straight lines, $ax^2 + 2hxy + by^2 = 0$ is $\dfrac{x^2 - y^2}{a - b} = \dfrac{xy}{h}$.

Solved Example # 35
Show that the equation 6x² – 5xy + y² = 0 represents a pair of distinct straight lines, each passing through the origin. Find the separate equations of these lines.

Solution.
The given equation is a homogeneous equation of second degree. So, it represents a pair of straight lines passing through the origin. Comparing the given equation with
$ax^2 + 2hxy + by^2 = 0$, we obtain $a = 6$, $b = 1$ and $2h = -5$.

$\therefore$ $h^2 - ab = \dfrac{25}{4} - 6 = \dfrac{1}{4} > 0 \Rightarrow h^2 > ab$

Hence, the given equation represents a pair of distinct lines passing through the origin.

Now, $6x^2 - 5xy + y^2 = 0 \Rightarrow \left(\dfrac{y}{x}\right)^2 \mathbf{- 5}\left(\dfrac{y}{x}\right) \mathbf{+ 6 = 0}$

$\Rightarrow \left(\dfrac{y}{x}\right)^2 - 3\left(\dfrac{y}{x}\right) - 2\left(\dfrac{y}{x}\right) + 6 = 0 \Rightarrow \left(\dfrac{y}{x} - 3\right)\left(\dfrac{y}{x} - 2\right) = 0$

$\Rightarrow \dfrac{y}{x} - 3 = 0$ or $\dfrac{y}{x} - 2 = 0 \Rightarrow y - 3x = 0$ or $y - 2x = 0$

So the given equation represents the straight lines **y – 3x = 0 and y – 2x = 0 Ans.**

Solved Example # 36 Find the equations to the pair of lines through the origin which are perpendicular to the lines represented by $2x^2 - 7xy + 2y^2 = 0$.

Solution.

We have $2x^2 - 7xy + 2y^2 = 0$.

$\Rightarrow \quad 2x^2 - 6xy - xy + 3y^2 = 0 \Rightarrow \quad 2x(x - 3y) - y(x - 3y) = 0$

$\Rightarrow \quad (x - 3y)(2x - y) = 0 \quad \Rightarrow \quad x - 3y = 0$ or $2x - y = 0$

Thus the given equation represents the lines $x - 3y = 0$ and $2x - y = 0$. The equations of the lines passing through the origin and perpendicular to the given lines are $y - 0 = -3(x - 0)$

and $y - 0 = -\frac{1}{2}(x - 0)$ [$\because$ (Slope of $x - 3y = 0$) is 1/3 and (Slope of $2x - y = 0$) is 2]

$\Rightarrow$ **$y + 3x = 0$ and $2y + x = 0$ Ans.**

Solved Example # 37

Find the angle between the pair of straight lines $4x^2 + 24xy + 11y^2 = 0$

Solution. Given equation is $4x^2 + 24xy + 11y^2 = 0$

Here a = coeff. of x^2 = 4, b = coeff. of y^2 = 11

and 2h = coeff. of xy = 24 $\quad \therefore \quad$ h = 12

Now $\tan\theta = \left|\dfrac{2\sqrt{h^2 - ab}}{a + b}\right| = \left|\dfrac{2\sqrt{144 - 44}}{4 + 11}\right| = \dfrac{4}{3}$

Where θ is the acute angle between the lines.

$\therefore$ acute angle between the lines is $\tan^{-1}\left(\frac{4}{3}\right)$ and obtuse angle between them is

$\pi - \tan^{-1}\left(\frac{4}{3}\right)$ Ans.

Solved Example # 38 Find the equation of the bisectors of the angle between the lines represented by $3x^2 - 5xy + y^2 = 0$

Solution.

Given equation is $3x^2 - 5xy + y^2 = 0$ ....(1)

comparing it with the equation $ax^2 + 2hxy + by^2 = 0$ ....(2)

we have a = 3, 2h = –5; and b = 4

Now the equation of the bisectors of the angle between the pair of lines (1) is $\dfrac{x^2 - y^2}{a - b} = \dfrac{xy}{h}$

or $\dfrac{x^2 - y^2}{3 - 4} = \dfrac{xy}{-\frac{5}{2}}$; $\quad$ or $\dfrac{x^2 - y^2}{-1} = \dfrac{2xy}{-5}$

or **$5x^2 - 2xy - 5y^2 = 0$ Ans.**

Self practice problems :

34. Find the area of the triangle formed by the lines $y^2 - 9xy + 18x^2 = 0$ and $y = 9$. **Ans.** $\frac{27}{4}$ sq. units

35. If the pairs of straight lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angle between the other pair, prove that $pq = -1$.

21. General equation of second degree representing a pair of Straight lines:

(i) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines if :

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$, i.e. if $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$.

(ii) The angle θ between the two lines representing by a general equation is the same as that between the two lines represented by its homogeneous part only.

Solved Example # 39 Prove that the equation $2x^2 + 5xy + 3y^2 + 6x + 7y + 4 = 0$ represents a pair of straight lines. Find the co-ordinates of their point of intersection and also the angle between them.

Solution. Given equation is

$$2x^2 + 5xy + 2y^2 + 6x + 7y + 4 = 0$$

Writing the equation (1) as a quadratic equation in x we have

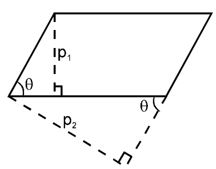
$$2x^2 + (5y + 6)x + 3y^2 + 7y + 4 = 0$$

$\therefore \quad x = \dfrac{-(5y + 6) \pm \sqrt{(5y + 6)^2 - 4.2(3y^2 + 7y + 4)}}{4}$

$= \dfrac{-(5y + 6) \pm \sqrt{25y^2 + 60y + 36 - 24y^2 - 56y - 32}}{4}$

$= \dfrac{-(5y + 6) \pm \sqrt{y^2 + 4y + 4}}{4} = \dfrac{-(5y + 6) \pm (y + 2)}{4}$

$\therefore \quad x = \dfrac{-5y - 6 + y + 2}{4}, \dfrac{-5y - 6 - y - 2}{4}$



or $4x + 4y + 4 = 0$ and $4x + 6y + 8 = 0$

or $x + y + 1 = 0$ and $2x + 3y + 4 = 0$

Hence equation (1) represents a pair of straight lines whose equation are **$x + y + 1 = 0$** .....(1)

and **$2x + 3y + 4 = 0$** ....(2) **Ans.**

Solving these two equations, the required point of intersection is **(1, – 2) Ans.**

Self practice problem :

36. Find the combined equation of the straight lines passing through the point (1, 1) and parallel to the lines represented by the equation $x^2 - 5xy + 4y^2 + x + 2y - 2 = 0$ and find the angle between them.

Ans. $x^2 - 5xy + 4y^2 + 3x - 3y = 0$, $\tan^{-1}\left(\frac{3}{5}\right)$

22. Homogenization :

The equation of a pair of straight lines joining origin to the points of intersection of the line $L \equiv \ell x + my + n = 0$ and a second degree curve,

$S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

is $ax^2 + 2hxy + by^2 + 2gx\left(\frac{\ell x + my}{-n}\right) + 2fy\left(\frac{\ell x + my}{-n}\right) + c\left(\frac{\ell x + my}{-n}\right)^2 = 0.$

The equation is obtained by homogenizing the equation of curve with the help of equation of line.

NOTE : Equation of any curve passing through the points of intersection of two curves $C_1 = 0$ and $C_2 = 0$ is given by $\lambda C_1 + \mu C_2 = 0$ where λ & μ are parameters.

Solved Example # 40

Prove that the angle between the lines joining the origin to the points of intersection of the straight line y = 3x + 2 with the curve $x^2 + 2xy + 3y^2 + 4x + 8y - 11 = 0$ is $\tan^{-1}\frac{2\sqrt{2}}{3}$.

Solution.

Equation of the given curve is $x^2 + 2xy + 3y^2 + 4x + 8y - 11 = 0$

and equation of the given straight line is $y - 3x = 2$; $\therefore$ $\frac{y-3x}{2} = 1$

Making equation (1) homogeneous equation of the second degree in x any y with the help of (1), we have

$$x^2 + 2xy + 3y^2 + 4x\left(\frac{y-3x}{2}\right) + 8y\left(\frac{y-3x}{2}\right) - 11\left(\frac{y-3x}{2}\right)^2 = 0$$

or $x^2 + 2xy + 3y^2 + \frac{1}{2}(4xy + 8y^2 - 12x^2 - 24xy) - \frac{11}{4}(y^2 - 6xy + 9x^2) = 0$

or $4x^2 + 8xy + 12y^2 + 2(8y^2 - 12x^2 - 20xy) - 11(y^2 - 6xy + 9x^2) = 0$

or $-119x^2 + 34xy + 17y^2 = 0$ or $119x^2 - 34xy - 17y^2 = 0$

or $7x^2 - 2xy - y^2 = 0$

This is the equation of the lines joining the origin to the points of intersection of (1) and (2).

Comparing equation (3) with the equation $ax^2 + 2hxy + by^2 = 0$

we have $a = 7$, $b = -1$ and $2h = -2$ i.e. $h = -1$

If θ be the acute angle between pair of lines (3), then

$$\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right| = \left|\frac{2\sqrt{1+7}}{7-1}\right| = \frac{2\sqrt{8}}{6} = \frac{2\sqrt{2}}{3} \quad \therefore \quad \theta = \tan^{-1}\frac{2\sqrt{2}}{3} \quad \textbf{Proved}$$

Self practice problems :

37. Find the equation of the straight lines joining the origin to the points of intersection of the line $3x + 4y - 5 = 0$ and the curve $2x^2 + 3y^2 = 5$.

Ans. $x^2 - y^2 - 24xy = 0$

38. Find the equation of the straight lines joining the origin to the points of intersection of the line $lx + my + n = 0$ and the curve $y^2 = 4ax$. Also, find the condition of their perpendicularity.

Ans. $4alx^2 + 4amxy + ny^2 = 0$; $4al + n = 0$

SHORT REVISION

1. **DISTANCE FORMULA :** The distance between the points $A(x_1,y_1)$ and $B(x_2,y_2)$ is $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$.

2. **SECTION FORMULA :** If $P(x , y)$ divides the line joining $A(x_1 , y_1)$ & $B(x_2 , y_2)$ in the ratio $m : n$, then ;

$x = \dfrac{mx_2+nx_1}{m+n}$; $y = \dfrac{my_2+ny_1}{m+n}$.

If $\frac{m}{n}$ is positive, the division is internal, but if $\frac{m}{n}$ is negative, the division is external .

Note : If P divides AB internally in the ratio m : n & Q divides AB externally in the ratio m : n then P & Q are said to be harmonic conjugate of each other w.r.t. AB.

Mathematically ; $\dfrac{2}{AB}=\dfrac{1}{AP}+\dfrac{1}{AQ}$ i.e. AP, AB & AQ are in H.P.

3. **CENTROID AND INCENTRE :** If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of triangle ABC, whose sides BC, CA, AB are of lengths a, b, c respectively, then the coordinates of the centroid are : $\left(\dfrac{x_1+x_2+x_3}{3},\dfrac{y_1+y_2+y_3}{3}\right)$

& the coordinates of the incentre are : $\left(\dfrac{ax_1+bx_2+cx_3}{a+b+c},\dfrac{ay_1+by_2+cy_3}{a+b+c}\right)$

Note that incentre divides the angle bisectors in the ratio $(b+c) : a$; $(c+a) : b$ & $(a+b) : c$.

REMEMBER :

(i) Orthocentre , Centroid & circumcentre are always collinear & centroid divides the line joining orthocentre & cercumcentre in the ratio 2 : 1 .

(ii) In an isosceles triangle G, O, I & C lie on the same line .

4. **SLOPE FORMULA :**

If θ is the angle at which a straight line is inclined to the positive direction of x–axis, & $0° \le \theta < 180°$, $\theta \ne 90°$, then the slope of the line, denoted by m, is defined by $m = \tan\theta$. If θ is 90°, m does not exist, but the line is parallel to the y–axis.

If $\theta = 0$, then $m = 0$ & the line is parallel to the x–axis.

If $A (x_1, y_1)$ & $B (x_2, y_2)$, $x_1 \ne x_2$, are points on a straight line, then the slope m of the line is given by: $m = \left(\dfrac{y_1-y_2}{x_1-x_2}\right)$.

5. **CONDITION OF COLLINEARITY OF THREE POINTS – (SLOPE FORM) :**

Points $A (x_1, y_1)$, $B (x_2, y_2)$, $C(x_3, y_3)$ are collinear if $\left(\dfrac{y_1-y_2}{x_1-x_2}\right)=\left(\dfrac{y_2-y_3}{x_2-x_3}\right)$.

6. **EQUATION OF A STRAIGHT LINE IN VARIOUS FORMS :**

(i) **Slope – intercept form:** $y = mx + c$ is the equation of a straight line whose slope is m & which makes an intercept c on the y–axis .

(ii) **Slope one point form:** $y - y_1 = m (x - x_1)$ is the equation of a straight line whose slope is m & which passes through the point (x_1, y_1).

(iii) **Parametric form :** The equation of the line in parametric form is given by $\dfrac{x-x_1}{\cos\theta}=\dfrac{y-y_1}{\sin\theta}= r$ (say). Where 'r' is the distance of any point (x , y) on the line from the fixed point (x_1, y_1) on the line. r is positive if the point (x, y) is on the right of (x_1, y_1) and negative if (x, y) lies on the left of (x_1, y_1) .

(iv) **Two point form :** $y - y_1 = \dfrac{y_2-y_1}{x_2-x_1}(x - x_1)$ is the equation of a straight line which passes through the points (x_1, y_1) & (x_2, y_2) .

(v) **Intercept form :** $\dfrac{x}{a}+\dfrac{y}{b} = 1$ is the equation of a straight line which makes intercepts a & b on OX & OY respectively .

(vi) **Perpendicular form :** $x\cos\alpha + y\sin\alpha = p$ is the equation of the straight line where the length of the perpendicular from the origin O on the line is p and this perpendicular makes angle α with positive side of x–axis .

(vii) **General Form :** $ax + by + c = 0$ is the equation of a straight line in the general form

7. **POSITION OF THE POINT (x_1, y_1) RELATIVE TO THE LINE $ax + by + c = 0$:**

If $ax_1 + by_1 + c$ is of the same sign as c, then the point (x_1, y_1) lie on the origin side of $ax + by + c = 0$. But if the sign of $ax_1 + by_1 + c$ is opposite to that of c, the point (x_1, y_1) will lie on the non-origin side of $ax + by + c = 0$.

8. **THE RATIO IN WHICH A GIVEN LINE DIVIDES THE LINE SEGMENT JOINING TWO POINTS :**

Let the given line $ax + by + c = 0$ divide the line segment joining $A(x_1, y_1)$ & $B(x_2, y_2)$ in the ratio m : n, then $\dfrac{m}{n}=-\dfrac{ax_1+by_1+c}{ax_2+by_2+c}$. If A & B are on the same side of the given line then $\frac{m}{n}$ is negative but if A & B are on opposite sides of the given line , then $\frac{m}{n}$ is positive

9. **LENGTH OF PERPENDICULAR FROM A POINT ON A LINE :**

The length of perpendicular from $P(x_1, y_1)$ on $ax + by + c = 0$ is $\left|\dfrac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$.

10. **ANGLE BETWEEN TWO STRAIGHT LINES IN TERMS OF THEIR SLOPES :**

If m_1 & m_2 are the slopes of two intersecting straight lines $(m_1 m_2 \ne -1)$ & θ is the acute angle between them, then

$\tan\theta = \left|\frac{m_1 - m_2}{1+m_1 m_2}\right|$.

Note : Let m_1, m_2, m_3 are the slopes of three lines $L_1 = 0$; $L_2 = 0$; $L_3 = 0$ where $m_1 > m_2 > m_3$ then the interior angles of the Δ ABC found by these lines are given by,

$\tan A = \frac{m_1 - m_2}{1 + m_1 m_2}$; $\tan B = \frac{m_2 - m_3}{1 + m_2 m_3}$ & $\tan C = \frac{m_3 - m_1}{1 + m_3 m_1}$

11. PARALLEL LINES :

(i) When two straight lines are parallel their slopes are equal. Thus any line parallel to $ax + by + c = 0$ is of the type $ax + by + k = 0$. Where k is a parameter.

(ii) The distance between two parallel lines with equations $ax + by + c_1 = 0$ & $ax + by + c_2 = 0$ is $\left|\frac{c_1 - c_2}{\sqrt{a^2+b^2}}\right|$.

Note that the coefficients of x & y in both the equations must be same.

(iii) The area of the parallelogram $= \frac{p_1 p_2}{\sin\theta}$, where p_1 & p_2 are distances between two pairs of opposite sides & θ is the angle between any two adjacent sides . Note that area of the parallelogram bounded by the lines $y = m_1x + c_1$, $y = m_1x + c_2$ and $y = m_2x + d_1$, $y = m_2x + d_2$ is given by $\left|\frac{(c_1 - c_2)(d_1 - d_2)}{m_1 - m_2}\right|$.

12. PERPENDICULAR LINES :

(i) When two lines of slopes m_1 & m_2 are at right angles, the product of their slopes is -1, i.e. $m_1 m_2 = -1$. Thus any line perpendicular to $ax + by + c = 0$ is of the form $bx - ay + k = 0$, where k is any parameter.

(ii) Straight lines $ax + by + c = 0$ & $a'x + b'y + c' = 0$ are at right angles if & only if $aa' + bb' = 0$.

13. Equations of straight lines through (x_1, y_1) making angle α with $y = mx + c$ are:
$(y - y_1) = \tan(\theta - \alpha)(x - x_1)$ & $(y - y_1) = \tan(\theta + \alpha)(x - x_1)$, where $\tan\theta = m$.

14. CONDITION OF CONCURRENCY :

Three lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ & $a_3x + b_3y + c_3 = 0$ are concurrent if

$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$. **Alternatively :** If three constants A, B & C can be found such that

$A(a_1x + b_1y + c_1) + B(a_2x + b_2y + c_2) + C(a_3x + b_3y + c_3) \equiv 0$, then the three straight lines are concurrent.

15. AREA OF A TRIANGLE :

If (x_i, y_i), i = 1, 2, 3 are the vertices of a triangle, then its area is equal to $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$, provided the vertices are considered in the counter clockwise sense. The above formula will give a (–) ve area if the vertices (x_i, y_i), i = 1, 2, 3 are placed in the clockwise sense.

16. CONDITION OF COLLINEARITY OF THREE POINTS–(AREA FORM):

The points (x_i, y_i), i = 1 , 2 , 3 are collinear if $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$.

17. THE EQUATION OF A FAMILY OF STRAIGHT LINES PASSING THROUGH THE POINTS OF INTERSECTION OF TWO GIVEN LINES:

The equation of a family of lines passing through the point of intersection of
$a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$ is given by $(a_1x + b_1y + c_1) + k(a_2x + b_2y + c_2) = 0$, where k is an arbitrary real number.

Note: If $u_1 = ax + by + c$, $u_2 = a'x + b'y + d$, $u_3 = ax + by + c'$, $u_4 = a'x + b'y + d'$
then, $u_1 = 0$; $u_2 = 0$; $u_3 = 0$; $u_4 = 0$ form a parallelogram.
$u_2 u_3 - u_1 u_4 = 0$ represents the diagonal BD.

Proof : Since it is the first degree equation in x & y it is a straight line. Secondly point B satisfies the equation because the co–ordinates of B satisfy $u_2 = 0$ and $u_1 = 0$.
Similarly for the point D. Hence the result.
On the similar lines $u_1u_2 - u_3u_4 = 0$ represents the diagonal AC.

Note: The diagonal AC is also given by $u_1 + \lambda u_4 = 0$ and $u_2 + \mu u_3 = 0$, if the two equations are identical for some λ and μ.
[For getting the values of λ & μ compare the coefficients of x, y & the constant terms] .

18. BISECTORS OF THE ANGLES BETWEEN TWO LINES :

(i) Equations of the bisectors of angles between the lines $ax + by + c = 0$ & $a'x + b'y + c' = 0$ $(ab' \neq a'b)$ are : $\frac{ax + by + c}{\sqrt{a^2+b^2}} = \pm \frac{a'x + b'y + c'}{\sqrt{a'^2+b'^2}}$

(ii) **To discriminate between the acute angle bisector & the obtuse angle bisector**
If θ be the angle between one of the lines & one of the bisectors, find $\tan\theta$.
If $|\tan\theta| < 1$, then $2\theta < 90°$ so that this bisector is the acute angle bisector .
If $|\tan\theta| > 1$, then we get the bisector to be the obtuse angle bisector .

(iii) To discriminate between the bisector of the angle containing the origin & that of the angle not containing the origin. Rewrite the equations , $ax + by + c = 0$ & $a'x + b'y + c' = 0$ such that the constant terms c, c' are positive. Then;
$\frac{ax + by + c}{\sqrt{a^2+b^2}} = + \frac{a'x + b'y + c'}{\sqrt{a'^2+b'^2}}$ gives the equation of the bisector of the angle containing the origin & $\frac{ax + by + c}{\sqrt{a^2+b^2}} = - \frac{a'x + b'y + c'}{\sqrt{a'^2+b'^2}}$ gives the equation of the bisector of the angle not containing the origin.

(iv) To discriminate between acute angle bisector & obtuse angle bisector proceed as follows Write $ax + by + c = 0$ & $a'x + b'y + c' = 0$ such that constant terms are positive .

If $aa' + bb' < 0$, then the angle between the lines that contains the origin is acute and the equation of the bisector of this acute angle is $\frac{ax+by+c}{\sqrt{a^2+b^2}} = + \frac{a'x + b'y + c'}{\sqrt{a'^2+b'^2}}$

therefore $\frac{ax + by + c}{\sqrt{a^2+b^2}} = - \frac{a'x + b'y + c'}{\sqrt{a'^2+b'^2}}$ is the equation of other bisector.

If, however, $aa' + bb' > 0$, then the angle between the lines that contains the origin is obtuse & the equation of the bisector of this obtuse angle is:

$\frac{ax + by + c}{\sqrt{a^2+b^2}} = + \frac{a'x + b'y + c'}{\sqrt{a'^2+b'^2}}$; therefore $\frac{ax + by + c}{\sqrt{a^2+b^2}} = - \frac{a'x + b'y + c'}{\sqrt{a'^2+b'^2}}$

is the equation of other bisector.

(v) Another way of identifying an acute and obtuse angle bisector is as follows :
Let $L_1 = 0$ & $L_2 = 0$ are the given lines & $u_1 = 0$ and $u_2 = 0$ are the bisectors between $L_1 = 0$ & $L_2 = 0$. Take a point P on any one of the lines $L_1 = 0$ or $L_2 = 0$ and drop perpendicular on $u_1 = 0$ & $u_2 = 0$ as shown. If ,

$|p| < |q| \Rightarrow u_1$ is the acute angle bisector .

$|p| > |q| \Rightarrow u_1$ is the obtuse angle bisector .

$|p| = |q| \Rightarrow$ the lines L_1 & L_2 are perpendicular .

Note : Equation of straight lines passing through $P(x_1, y_1)$ & equally inclined with the lines $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$ are those which are parallel to the bisectors between these two lines & passing through the point P.

19. A PAIR OF STRAIGHT LINES THROUGH ORIGIN :

(i) A homogeneous equation of degree two of the type $ax^2 + 2hxy + by^2 = 0$ always represents a pair of straight lines passing through the origin & if :

(a) $h^2 > ab \Rightarrow$ lines are real & distinct .
(b) $h^2 = ab \Rightarrow$ lines are coincident .
(c) $h^2 < ab \Rightarrow$ lines are imaginary with real point of intersection i.e. (0, 0)

(ii) If $y = m_1x$ & $y = m_2x$ be the two equations represented by $ax^2 + 2hxy + by^2 = 0$, then;

$m_1 + m_2 = -\frac{2h}{b}$ & $m_1 m_2 = \frac{a}{b}$.

(iii) If θ is the acute angle between the pair of straight lines represented by,

$ax^2 + 2hxy + by^2 = 0$, then; $\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$.

The condition that these lines are:
(a) At right angles to each other is $a + b = 0$. i.e. co–efficient of x^2 + coefficient of $y^2 = 0$.
(b) Coincident is $h^2 = ab$. **(c)** Equally inclined to the axis of x is $h = 0$. i.e. coeff. of $xy = 0$.

Note: A homogeneous equation of degree n represents n straight lines passing through origin.

20. GENERAL EQUATION OF SECOND DEGREE REPRESENTING A PAIR OF STRAIGHT LINES:

(i) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines if:

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$, i.e. if $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$.

(ii) The angle θ between the two lines representing by a general equation is the same as that between the two lines represented by its homogeneous part only .

21. The joint equation of a pair of straight lines joining origin to the points of intersection of the line given by $lx + my + n = 0$ .... **(i)** &
the 2nd degree curve : $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ .... **(ii)**

is $ax^2 + 2hxy + by^2 + 2gx\left(\frac{lx+my}{-n}\right) + 2fy\left(\frac{lx+my}{-n}\right) + c\left(\frac{lx+my}{-n}\right)^2 = 0$ ...... **(iii)**

(iii) is obtained by homogenizing (ii) with the help of **(i)**, by writing **(i)** in the form: $\left(\frac{lx+my}{-n}\right) = 1$.

22. The equation to the straight lines bisecting the angle between the straight lines,

$ax^2 + 2hxy + by^2 = 0$ is $\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$.

23. The product of the perpendiculars, dropped from (x_1, y_1) to the pair of lines represented by the equation, $ax^2 + 2hxy + by^2 = 0$ is $\frac{ax_1^2 + 2hx_1y_1 + by_1^2}{\sqrt{(a-b)^2 + 4h^2}}$.

24. Any second degree curve through the four point of intersection of $f(x\,y) = 0$ & $xy = 0$ is given by $f(x\,y) + \lambda xy = 0$ where $f(xy) = 0$ is also a second degree curve.

EXERCISE–1

Q.1 The sides AB, BC, CD, DA of a quadrilateral have the equations $x + 2y = 3$, $x = 1$, $x - 3y = 4$, $5x + y + 12 = 0$ respectively. Find the angle between the diagonals AC & BD.

Q.2 Find the co-ordinates of the orthocentre of the triangle, the equations of whose sides are $x + y = 1$, $2x + 3y = 6$, $4x - y + 4 = 0$, without finding the co–ordinates of its vertices.

Q.3 Two vertices of a triangle are (4, –3) & (–2, 5). If the orthocentre of the triangle is at (1, 2), find the coordinates of the third vertex.

Q.4 The point A divides the join of P(–5, 1) & Q(3, 5) in the ratio K : 1 . Find the two values of K for which the area of triangle ABC, where B is (1, 5) & C is (7, –2), is equal to 2 units in magnitude.

Q.5 Determine the ratio in which the point P(3, 5) divides the join of A(1, 3) & B(7, 9). Find the harmonic conjugate of P w.r.t. A & B.

Q.6 A line is such that its segment between the straight lines $5x - y - 4 = 0$ and $3x + 4y - 4 = 0$ is bisected at the point (1, 5). Obtain the equation.

Q.7 A line through the point P(2, –3) meets the lines $x - 2y + 7 = 0$ and $x + 3y - 3 = 0$ at the points A and B respectively. If P divides AB externally in the ratio 3 : 2 then find the equation of the line AB.

Q.8 The area of a triangle is 5. Two of its vertices are (2, 1) & (3, –2). The third vertex lies on $y = x + 3$. Find the third vertex.

Q.9 A variable line, drawn through the point of intersection of the straight lines $\frac{x}{a}+\frac{y}{b} = 1$ & $\frac{x}{b}+\frac{y}{a} = 1$, meets the coordinate axes in A & B . Show that the locus of the mid point of AB is the curve $2xy(a + b) = ab(x + y)$.

Q.10 Two consecutive sides of a parallelogram are $4x + 5y = 0$ & $7x + 2y = 0$. If the equation to one diagonal is $11x + 7y = 9$, find the equation to the other diagonal.

Q.11 The line $3x + 2y = 24$ meets the y–axis at A & the x–axis at B. The perpendicular bisector of AB meets the line through (0, –1) parallel to x–axis at C. Find the area of the triangle ABC.

Q.12 If the straight line drawn through the point $P(\sqrt{3}, 2)$ & making an angle $\frac{\pi}{6}$ with the x–axis, meets the line $\sqrt{3}x - 4y + 8 = 0$ at Q. Find the length PQ.

Q.13 Find the condition that the diagonals of the parallelogram formed by the lines $ax + by + c = 0$; $ax + by + c' = 0$; $a'x + b'y + c = 0$ & $a'x + b'y + c' = 0$ are at right angles. Also find the equation to the diagonals of the parallelogram.

Q.14 If lines be drawn parallel to the axes of co-ordinates from the points where $x \cos\alpha + y \sin\alpha = p$ meets them so as to meet the perpendicular on this line from the origin in the points P and Q then prove that $| PQ | = 4p\, | \cos2\alpha |\, \text{cosec}^2 2\alpha$.

Q.15 The points (1, 3) & (5, 1) are two opposite vertices of a rectangle. The other two vertices lie on the line $y = 2x + c$. Find c & the remaining vertices.

Q.16 A straight line L is perpendicular to the line $5x - y = 1$. The area of the triangle formed by the line L & the coordinate axes is 5. Find the equation of the line.

Q.17 Two equal sides of an isosceles triangle are given by the equations $7x - y + 3 = 0$ and $x + y - 3 = 0$ & its third side passes through the point (1, –10). Determine the equation of the third side.

Q.18 The vertices of a triangle OBC are O (0, 0), B (–3, – 1), C(–1, –3). Find the equation of the line parallel to BC & intersecting the sides OB & OC, whose perpendicular distance from the point (0, 0) is half.

Q.19 Find the direction in which a straight line may be drawn through the point (2, 1) so that its point of intersection with the line $4y - 4x + 4 + 3\sqrt{2} + 3\sqrt{10} = 0$ is at a distance of 3 units from (2, 1).

Q.20 Consider the family of lines, $5x + 3y - 2 + K_1 (3x - y - 4) = 0$ and $x - y + 1 + K_2(2x - y - 2) = 0$. Find the equation of the line belonging to both the families without determining their vertices.

Q.21 Given vertices A (1, 1), B (4, –2) & C (5, 5) of a triangle, find the equation of the perpendicular dropped from C to the interior bisector of the angle A.

Q.22 If through the angular points of a triangle straight lines be drawn parallel to the opposite sides, and if the intersections of these lines be joined to the opposite angular points of the traingle then using co-ordinate geometry, show that the lines so obtained are concurrent.

Q.23 Determine all values of α for which the point (α, α²) lies inside the triangle formed by the lines $2x + 3y - 1 = 0$; $x + 2y - 3 = 0$; $5x - 6y - 1 = 0$.

Q.24 If the equation, $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represent a pair of straight lines, prove that the equation to the third pair of straight lines passing through the points where these meet the axes is, $ax^2 - 2hxy + by^2 + 2gx + 2fy + c + \frac{4fg}{c}xy = 0$.

Q.25 A straight line is drawn from the point (1, 0) to the curve $x^2 + y^2 + 6x - 10y + 1 = 0$, such that the intercept made on it by the curve subtends a right angle at the origin. Find the equations of the line.

Q.26 Determine the range of values of $\theta \in [0, 2\pi]$ for which the point $(\cos\theta, \sin\theta)$ lies inside the triangle formed by the lines $x + y = 2$; $x - y = 1$ & $6x + 2y - \sqrt{10} = 0$.

Q.27 Find the co-ordinates of the incentre of the triangle formed by the line $x + y + 1 = 0$; $x - y + 3 = 0$ & $7x - y + 3 = 0$. Also find the centre of the circle escribed to $7x - y + 3 = 0$.

Q.28 In a triangle ABC, D is a point on BC such that $\frac{BD}{DC}=\frac{AB}{AC}$. The equation of the line AD is $2x + 3y + 4 = 0$ & the equation of the line AB is $3x + 2y + 1 = 0$. Find the equation of the line AC.

Q.29 Show that all the chords of the curve $3x^2 - y^2 - 2x + 4y = 0$ which subtend a right angle at the origin are concurrent. Does this result also hold for the curve, $3x^2 + 3y^2 - 2x + 4y = 0$? If yes, what is the point of concurrency & if not, give reasons.

Q.30 Without finding the vertices or angles of the triangle, show that the three straight lines $au + bv = 0$; $au - bv = 2ab$ and $u + b = 0$ from an isosceles triangle where $u \equiv x + y - b$ & $v \equiv x - y - a$ & $a, b \neq 0$.

EXERCISE–2

Q.1 The equations of perpendiculars of the sides AB & AC of triangle ABC are $x - y - 4 = 0$ and $2x - y - 5 = 0$ respectively. If the vertex A is (– 2, 3) and point of intersection of perpendiculars bisectors is $\left(\frac{3}{2}, \frac{5}{2}\right)$, find the equation of medians to the sides AB & AC respectively.

Q.2 A line $4x + y = 1$ through the point A(2, –7) meets the line BC whose equation is $3x - 4y + 1 = 0$ at a point B. Find the equation of the line AC, so that AB = AC.

Q.3 If $x \cos\alpha + y \sin\alpha = p$, where $p = -\frac{\sin^2\alpha}{\cos\alpha}$ be a straight line, prove that the perpendiculars on this straight line from the points (m², 2m), (mm′, m + m′), (m′², 2m′) form a G.P.

Q.4 A(3, 0) and B(6, 0) are two fixed points and $P(x_1, y_1)$ is a variable point. AP and BP meet the y-axis at C & D respectively and AD meets OP at Q where 'O' is the origin. Prove that CQ passes through a fixed point and find its co–ordinates.

Q.5 Find the equation of the straight lines passing through (–2, –7) & having an intercept of length 3 between the straight lines $4x + 3y = 12$, $4x + 3y = 3$.

Q.6 Let ABC be a triangle with AB = AC. If D is the mid point of BC, E the foot of the perpendicular from D to AC and F the midpoint of DE, prove analytically that AF is perpendicular to BE.

Q.7 Two sides of a rhombous ABCD are parallel to the lines $y = x + 2$ & $y = 7x + 3$. If the diagonals of the rhombous intersect at the point (1, 2) & the vertex A is on the y-axis, find the possible coordinates of A.

Q.8 The equations of the perpendicular bisectors of the sides AB & AC of a triangle ABC are $x - y + 5 = 0$ & $x + 2y = 0$, respectively. If the point A is $(1, -2)$, find the equation of the line BC.

Q.9 A pair of straight lines are drawn through the origin form with the line $2x + 3y = 6$ an isosceles triangle right angled at the origin. Find the equation of the pair of straight lines & the area of the triangle correct to two places of decimals.

Q.10 A triangle is formed by the lines whose equations are AB : $x + y - 5 = 0$, BC : $x + 7y - 7 = 0$ and CA : $7x + y + 14 = 0$. Find the bisector of the interior angle at B and the exterior angle at C. Determine the nature of the interior angle at A and find the equaion of the bisector.

Q.11 A point P is such that its perpendicular distance from the line $y - 2x + 1 = 0$ is equal to its distance from the origin. Find the equation of the locus of the point P. Prove that the line $y = 2x$ meets the locus in two points Q & R, such that the origin is the mid point of QR.

Q.12 A triangle has two sides $y = m_1x$ and $y = m_2x$ where m_1 and m_2 are the roots of the equation $b\alpha^2 + 2h\alpha + a = 0$. If (a, b) be the orthocentre of the triangle, then find the equation of the third side in terms of a, b and h.

Q.13 Find the area of the triangle formed by the straight lines whose equations are $x + 2y - 5 = 0$; $2x + y - 7 = 0$ and $x - y + 1 = 0$ without determining the coordinates of the vertices of the triangle. Also compute the tangent of the interior angles of the triangle and hence comment upon the nature of triangle.

Q.14 Find the equation of the two straight lines which together with those given by the equation $6x^2 - xy - y^2 + x + 12y - 35 = 0$ will make a parallelogram whose diagonals intersect in the origin.

Q.15 Find the equations of the sides of a triangle having $(4, -1)$ as a vertex, if the lines $x - 1 = 0$ and $x - y - 1 = 0$ are the equations of two internal bisectors of its angles.

Q.16 Equation of a line is given by $y + 2at = t(x - at^2)$, t being the parameter. Find the locus of the point of intersection of the lines which are at right angles.

Q.17 The ends A, B of a straight line line segment of a constant length 'c' slide upon the fixed rectangular axes OX & OY respectively. If the rectangle OAPB be completed then show that the locus of the foot of the perpendicular drawn from P to AB is $x^{2/3} + y^{2/3} = c^{2/3}$.

Q.18 A point moves so that the distance between the feet of the perpendiculars from it on the lines $bx^2 + 2hxy + ay^2 = 0$ is a constant 2d. Show that the equation to its locus is,
$$(x^2 + y^2)(h^2 - ab) = d^2\{(a - b)^2 + 4h^2\}$$

Q.19 The sides of a triangle are $U_r \equiv x\cos\alpha_r + y\sin\alpha_r - p_r = 0$, $(r = 1, 2, 3)$. Show that the orthocentre is given by $U_1\cos(\alpha_2 - \alpha_3) = U_2\cos(\alpha_3 - \alpha_1) = U_3\cos(\alpha_1 - \alpha_2)$.

Q.20 P is the point $(-1, 2)$, a variable line through P cuts the x & y axes at A & B respectively Q is the point on AB such that PA, PQ, PB are H.P. Show that the locus of Q is the line $y = 2x$.

Q.21 The equations of the altitudes AD, BE, CF of a triangle ABC are $x + y = 0$, $x - 4y = 0$ and $2x - y = 0$ respectively. The coordinates of A are $(t, -t)$. Find coordinates of B & C. Prove that if t varies the locus of the centroid of the triangle ABC is $x + 5y = 0$.

Q.22 A variable line is drawn through O to cut two fixed straight lines L_1 & L_2 in R & S. A point P is chosen on the variable line such that; $\frac{m+n}{OP} = \frac{m}{OR} + \frac{n}{OS}$. Show that the locus of P is a straight line passing the point of intersection of L_1 & L_2.

Q.23 If the lines $ax^2 + 2hxy + by^2 = 0$ from two sides of a parallelogram and the line $lx + my = 1$ is one diagonal, prove that the equation of the other diagonal is, $y(bl - hm) = x(am - hl)$

Q.24 The distance of a point (x_1, y_1) from each of two straight lines which passes through the origin of co-ordinates is δ; find the combined equation of these straight lines.

Q.25 The base of a triangle passes through a fixed point (f, g) & its sides are respectively bisected at right angles by the lines $y^2 - 8xy - 9x^2 = 0$. Determine the locus of its vertex.

EXERCISE–3

Q.1 The graph of the function, $\cos x \cos(x + 2) - \cos^2(x + 1)$ is:
(A) a straight line passing through $(0, -\sin^2 1)$ with slope 2
(B) a straight line passing through (0, 0)
(C) a parabola with vertex $(1, -\sin^2 1)$
(D) a straight line passing through the point $\left(\frac{\pi}{2}, -\sin^2 1\right)$ & parallel to the x–axis. [JEE '97, 2]

Q.2 One diagonal of a square is the portion of the line $7x + 5y = 35$ intercepted by the axes, obtain the extremities of the other diagonal. [REE '97, 6]

Q.3 A variable line L passing through the point B (2, 5) intersects the line $2x^2 - 5xy + 2y^2 = 0$ at P & Q. Find the locus of the point R on L such that distances BP, BR & BQ are in harmonic progression. [REE '98, 6]

Q.4**(i)** Select the correct alternative(s) : [JEE '98, 2 x 3 = 6]

(a) If P (1, 2), Q (4, 6), R (5, 7) & S (a, b) are the vertices of a parallelogram PQRS, then :
(A) a = 2, b = 4 (B) a = 3, b = 4 (C) a = 2, b = 3 (D) a = 3, b = 5

(b) The diagonals of a parallelogram PQRS are along the lines $x + 3y = 4$ and $6x - 2y = 7$. Then PQRS must be a :
(A) rectangle (B) square (C) cyclic quadrilateral (D) rhombus

(c) If the vertices P, Q, R of a triangle PQR are rational points, which of the following points of the triangle PQR is/are always rational point(s)?
(A) centriod (B) incentre (C) circumcentre (D) orthocentre

(ii) Using coordinate geometry, prove that the three altitudes of any triangle are concurrent. [JEE '98, 8]

Q.5 The equation of two equal sides AB and AC of an isosceles triangle ABC are $x + y = 5$ & $7x - y = 3$ respectively. Find the equations of the side BC if the area of the triangle of ABC is 5 units. [REE '99, 6]

Q.6 Let PQR be a right angled isosceles triangle, right angled at P (2, 1). If the equation of the line QR is $2x + y = 3$, then the equation representing the pair of lines PQ and PR is
(A) $3x^2 - 3y^2 + 8xy + 20x + 10y + 25 = 0$ (B) $3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$
(C) $3x^2 - 3y^2 + 8xy + 10x + 15y + 20 = 0$ (D) $3x^2 - 3y^2 - 8xy - 10x - 15y - 20 = 0$ [JEE'99, (2 out of 200)]

Q.7 (a) The incentre of the triangle with vertices $(1, \sqrt{3})$, (0, 0) and (2, 0) is :
(A) $\left(1, \frac{\sqrt{3}}{2}\right)$ (B) $\left(\frac{2}{3}, \frac{1}{\sqrt{3}}\right)$ (C) $\left(\frac{2}{3}, \frac{\sqrt{3}}{2}\right)$ (D) $\left(1, \frac{1}{\sqrt{3}}\right)$

(b) Let PS be the median of the triangle with vertices, P (2, 2), Q (6, -1) and R (7, 3). The equation of the line passing through (1, – 1) and parallel to PS is :

(A) $2x - 9y - 7 = 0$ (B) $2x - 9y - 11 = 0$
(C) $2x + 9y - 11 = 0$ (D) $2x + 9y + 7 = 0$
[JEE 2000 (Screening) 1 + 1 out of 35]

(c) For points $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ of the co-ordinate plane, a new distance d(P, Q) is defined by $d(P, Q) = |x_1 - x_2| + |y_1 - y_2|$. Let O = (0, 0) and A = (3, 2). Prove that the set of points in the first quadrant which are equidistant (with respect to the new distance) from O and A consists of the union of a line segment of finite length and an infinite ray. Sketch this set in a labelled diagram. [JEE 2000 (Mains) 10 out of 100]

Q.8 Find the position of point (4, 1) after it undergoes the following transformations successively.
(i) Reflection about the line, $y = x - 1$ (ii) Translation by one unit along x–axis in the positive direction .
(iii) Rotation through an angle $\frac{\pi}{4}$ about the origin in the anti–clockwise direction.
[REE 2000 (Mains) 3 out of 100]

Q.9 Area of the parallelogram formed by the lines y = mx, y = mx + 1, y = nx and y = nx + 1 equals
(A) $\frac{|m+n|}{(m-n)^2}$ (B) $\frac{2}{|m+n|}$ (C) $\frac{1}{|m+n|}$ (D) $\frac{1}{|m-n|}$ [JEE 2001 (Screening)]

Q.10 (a) Let P = (–1, 0), Q = (0, 0) and R = $(3, 3\sqrt{3})$ be three points. Then the equation of the bisector of the angle PQR is
(A) $\frac{\sqrt{3}}{2}x + y = 0$ (B) $x + \sqrt{3}y = 0$ (C) $\sqrt{3}x + y = 0$ (D) $x + \frac{\sqrt{3}}{2}y = 0$

(b) A straight line through the origin O meets the parallel lines 4x + 2y = 9 and 2x + y + 6 = 0 at points P and Q respectively. Then the point O divides the segment PQ in the ratio
(A) 1 : 2 (B) 3 : 4 (C) 2 : 1 (D) 4 : 3

(c) The area bounded by the curves y = |x| – 1 and y = –|x| + 1 is
(A) 1 (B) 2 (C) $2\sqrt{2}$ (D) 4
[JEE 2002 (Screening)]

(d) A straight line L through the origin meets the line x + y = 1 and x + y = 3 at P and Q respectively. Through P and Q two straight lines L_1 and L_2 are drawn, parallel to 2x – y = 5 and 3x + y = 5 respectively. Lines L_1 and L_2 intersect at R. Show that the locus of R, as L varies, is a straight line.
[JEE 2002 (Mains)]

Q.11 The area bounded by the angle bisectors of the lines $x^2 - y^2 + 2y = 1$ and the line x + y = 3, is
(A) 2 (B) 3 (C) 4 (D) 6 [JEE 2004 (Screening)]

Q.12 The area of the triangle formed by the intersection of a line parallel to x-axis and passing through P (h, k) with the lines y = x and x + y = 2 is $4h^2$. Find the locus of the point P. [JEE 2005, Mains, 2]

EXERCISE–4

Part : (A) Only one correct option

1. The equation of the internal bisector of ∠BAC of ΔABC with vertices A(5, 2), B(2, 3) and C(6, 5) is
(A) 2x + y + 12 = 0 (B) x + 2y – 12 = 0 (C) 2x + y – 12 = 0 (D) none of these

2. The lines ax + by + c = 0, where 3a + 2b + 4c = 0 are concurrent at the point :
(A) $\left(\frac{1}{2}, \frac{3}{4}\right)$ (B) (1, 3) (C) (3, 1) (D) $\left(\frac{3}{4}, \frac{1}{2}\right)$

3. The equation of second degree $x^2 + 2\sqrt{2}\,xy + 2y^2 + 4x + 4\sqrt{2}\,y + 1 = 0$ represents a pair of straight lines. The distance between them is
(A) 4 (B) $\frac{4}{\sqrt{3}}$ (C) 2 (D) $2\sqrt{3}$

4. The straight lines joining the origin to the points of intersection of the line 2x + y = 1 and curve $3x^2 + 4xy - 4x + 1 = 0$ include an angle :
(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

5. Given the points A (0, 4) and B (0, – 4), the equation of the locus of the point P (x, y) such that $|AP - BP| = 6$ is :
(A) $9x^2 - 7y^2 + 63 = 0$ (B) $9x^2 - 7y^2 - 63 = 0$
(C) $7x^2 - 9y^2 + 63 = 0$ (D) $7x^2 - 9y^2 - 63 = 0$

6. A triangle ABC with vertices A (– 1, 0), B (– 2, 3/4) & C (– 3, – 7/6) has its orthocentre H. Then the orthocentre of triangle BCH will be :
(A) (– 3, – 2) (B) (1, 3) (C) (– 1, 2) (D) none of these

7. Equation of a straight line passing through the origin and making with x – axis an angle twice the size of the angle made by the line y = 0.2 x with the x – axis, is :
(A) y = 0.4 x (B) y = (5/12) x (C) 6y – 5x = 0 (D) none of these

8. A variable straight line passes through a fixed point (a, b) intersecting the co–ordinates axes at A & B. If 'O' is the origin then the locus of the centroid of the triangle OAB is :
(A) bx + ay – 3xy = 0 (B) bx + ay – 2xy = 0
(C) ax + by – 3xy = 0 (D) ax + by – 2xy = 0

9. Area of the quadrilateral formed by the lines $|x| + |y| = 2$ is :
(A) 8 (B) 6 (C) 4 (D) none

10. The distance of the point (2, 3) from the line 2 x – 3 y + 9 = 0 measured along a line x - y + 1 = 0 is :
(A) $5\sqrt{3}$ (B) $4\sqrt{2}$ (C) $3\sqrt{2}$ (D) $2\sqrt{2}$

11. The set of values of 'b' for which the origin and the point (1, 1) lie on the same side of the straight line, $a^2x + a\,by + 1 = 0$ ∀ a ∈ R, b > 0 are :
(A) b ∈ (2, 4) (B) b ∈ (0, 2) (C) b ∈ [0, 2] (D) (2, ∞)

12. Drawn from the origin are two mutually perpendicular straight lines forming an isosceles triangle together with the straight line, 2x + y = a. Then the area of the triangle is :

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(A) $\frac{a^2}{2}$ (B) $\frac{a^2}{3}$ (C) $\frac{a^2}{5}$ (D) none

13. The line joining two points A (2, 0); B (3, 1) is rotated about A in the anticlock wise direction through an angle of 15º. The equation of the line in the new position is :
(A) $x - \sqrt{3}\,y - 2 = 0$ (B) $x - 2y - 2 = 0$
(C) $\sqrt{3}\,x - y - 2\sqrt{3} = 0$ (D) none

14. The line $x + 3y - 2 = 0$ bisects the angle between a pair of straight lines of which one has equation $x - 7y + 5 = 0$. The equation of the other line is :
(A) $3x + 3y - 1 = 0$ (B) $x - 3y + 2 = 0$ (C) $5x + 5y - 3 = 0$ (D) none

15. On the portion of the straight line, $x + 2y = 4$ intercepted between the axes, a square is constructed on the side of the line away from the origin. Then the point of intersection of its diagonals has co–ordinates :
(A) (2, 3) (B) (3, 2) (C) (3, 3) (D) none

16. A light beam emanating from the point A(3, 10) reflects from the straight line $2x + y - 6 = 0$ and then passes through the point B(4, 3). The equation of the reflected beam is :
(A) $3x - y + 1 = 0$ (B) $x + 3y - 13 = 0$ (C) $3x + y - 15 = 0$ (D) $x - 3y + 5 = 0$

17. The equation of the bisector of the angle between two lines $3x - 4y + 12 = 0$ and $12x - 5y + 7 = 0$ which contains the points $(-1, 4)$ is :
(A) $21x + 27y - 121 = 0$ (B) $21x - 27y + 121 = 0$
(C) $21x + 27y + 191 = 0$ (D) $\frac{-3x+4y-12}{5} = \frac{12x-5y+7}{13}$

18. The equation of bisectors of two lines L_1 & L_2 are $2x - 16y - 5 = 0$ and $64x + 8y + 35 = 0$. If the line L_1 passes through $(-11, 4)$, the equation of acute angle bisector of L_1 & L_2 is :
(A) $2x - 16y - 5 = 0$ (B) $64x + 8y + 35 = 0$ (C) data insufficient (D) none of these

19. The equation of the pair of bisectors of the angles between two straight lines is, $12x^2 - 7xy - 12y^2 = 0$. If the equation of one line is $2y - x = 0$ then the equation of the other line is :
(A) $41x - 38y = 0$ (B) $38x - 41y = 0$ (C) $38x + 41y = 0$ (D) $41x + 38y = 0$

20. If the straight lines joining the origin and the points of intersection of the curve $5x^2 + 12xy - 6y^2 + 4x - 2y + 3 = 0$ and $x + ky - 1 = 0$ are equally inclined to the x-axis then the value of k is equal to :
(A) 1 (B) -1 (C) 2 (D) 3

21. If the points of intersection of curves $C_1 = \lambda x^2 + 4y^2 - 2xy - 9x + 3$ & $C_2 = 2x^2 + 3y^2 - 4xy + 3x - 1$ subtends a right angle at origin, then the value of λ is :
(A) 19 (B) 9 (C) -19 (D) -9

Part : (B) May have more than one options correct

22. The equation of the bisectors of the angle between the two intersecting lines : $\frac{x-3}{\cos\theta} = \frac{y+5}{\sin\theta}$ and $\frac{x-3}{\cos\phi} = \frac{y+5}{\sin\phi}$ are $\frac{x-3}{\cos\alpha} = \frac{y+5}{\sin\alpha}$ and $\frac{x-3}{\beta} = \frac{y+5}{\gamma}$ then
(A) $\alpha = \frac{\theta+\phi}{2}$ (B) $\beta = -\sin\alpha$ (C) $\gamma = \cos\alpha$ (D) $\beta = \sin\alpha$

23. Equation of a straight line passing through the point of intersection of $x - y + 1 = 0$ and $3x + y - 5 = 0$ are perpendicular to one of them is
(A) $x + y + 3 = 0$ (B) $x + y - 3 = 0$ (C) $x - 3y - 5 = 0$ (D) $x - 3y + 5 = 0$

24. Three lines $px + qy + r = 0$, $qx + ry + p = 0$ and $rx + py + q = 0$ are concurrent if
(A) $p + q + r = 0$ (B) $p^2 + q^2 + r^2 = pq + qr + rp$
(C) $p^3 + p^3 + r^3 = 3pqr$ (D) none of these

25. Equation of a straight line passing through the point (4, 5) and equally inclined to the lines, $3x = 4y + 7$ and $5y = 12x + 6$ is
(A) $9x - 7y = 1$ (B) $9x + 7y = 71$ (C) $7x + 9y = 73$ (D) $7x - 9y + 17 = 0$

26. If the equation, $2x^2 + kxy - 3y^2 - x - 4y - 1 = 0$ represents a pair of lines then the value of k can be:
(A) 1 (B) 5 (C) -1 (D) -5

27. If $a^2 + 9b^2 - 4c^2 = 6ab$ then the family of lines $ax + by + c = 0$ are concurrent at :
(A) (1/2, 3/2) (B) (−1/2, −3/2) (C) (−1/2, 3/2) (D) (1/2, −3/2)

EXERCISE–5

1. If the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) be collinear, show that $\frac{y_2-y_3}{x_2x_3} + \frac{y_3-y_1}{x_3x_1} + \frac{y_1-y_2}{x_1x_2} = 0$.

2. Find the length of the perpendicular from the origin upon the straight line joining the two points whose coordinates are $(a\cos\alpha, a\sin\alpha)$ and $(a\cos\beta, a\sin\beta)$.

3. Show that the product of the perpendiculars drawn from the two points $(\pm\sqrt{a^2-b^2}, 0)$ upon the straight line $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ is b^2.

4. Find the equation of the bisector of the acute angle between the lines $3x - 4y + 7 = 0$ and $12x + 5y - 2 = 0$.

5. Find the equation to the pair of straight lines joining the origin to the intersections of the straight line $y = mx + c$ and the curve $x^2 + y^2 = a^2$. Prove that they are at right angles if $2c^2 = a^2(1 + m^2)$.

6. The variable line $x\cos\theta + y\sin\theta = 2$ cuts the x and y axes at A and B respectively. Find the locus of the vertex P of the rectangle OAPB, O being the origin.

7. If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of the triangle then show that :

(i) The median through A can be written in the form $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$.

(ii) the line through A & parallel to BC can be written in the form ; $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} - \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$.

(iii) equation to the angle bisector through A is $b\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + c\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$.

where b = AC & c = AB.

8. Is there a real value of λ for which the image of the point $(\lambda, \lambda - 1)$ by the line mirror $3x + y = 6\lambda$ is the point $(\lambda^2 + 1, \lambda)$? If so find λ.

9. If the straight lines, $ax + by + p = 0$ & $x\cos\alpha + y\sin\alpha - p = 0$ enclose an angle $\pi/4$ between them, and meet the straight line $x\sin\alpha - y\cos\alpha = 0$ in the same point, then find the value of $a^2 + b^2$.

10. Drive the conditions to be imposed on β so that $(0, \beta)$ should lie on or inside the triangle having sides $y + 3x + 2 = 0$, $3y - 2x - 5 = 0$ & $4y + x - 14 = 0$.

11. A straight line L is perpendicular to the line $5x - y = 1$. The area of the triangle formed by the line L & the coordinate axes is 5. Find the equation of the line.

12. Two equal sides of an isosceles triangle are given by the equations $7x - y + 3 = 0$ and $x + y - 3 = 0$ and its third side passes through the point (1, –10). Determine the equation of the third side.

13. Find the equations of the straight lines passing through the point (1, 1) and parallel to the lines represented by the equation, $x^2 - 5xy + 4y^2 + x + 2y - 2 = 0$.

14. Find the coordinates of the vertices of a square inscribed in the triangle with vertices A (0, 0), B (2, 1), C (3, 0); given that two of its vertices are on the side AC.

15. The equations of perpendiculars of the sides AB & AC of Δ ABC are $x - y - 4 = 0$ and $2x - y - 5 = 0$ respectively. If the vertex A is $(-2, 3)$ and point of intersection of perpendiculars bisectors $\left(\frac{3}{2}, \frac{5}{2}\right)$ is, find the equation of medians to the sides AB and AC respectively.

16. The sides of a triangle are $4x + 3y + 7 = 0$, $5x + 12y = 27$ and $3x + 4y + 8 = 0$. Find the equations of the internal bisectors of the angles and show that they are concurrent.

17. A ray of light is sent along the line $x - 2y - 3 = 0$. Upon reaching the line $3x - 2y - 5 = 0$, the ray is reflected from it. Find the equation of the line containing the reflected ray.

18. A triangle is formed by the lines whose equations are AB : $x + y - 5 = 0$, BC : $x + 7y - 7 = 0$ and CA : $7x + y + 14 = 0$. Find the bisector of the interior angle at B and the exterior angle at C. Determine the nature of the interior angle at A and find the equation of the bisector.

19. Find the equations of the sides of a triangle having (4, –1) as a vertex, if the lines $x - 1 = 0$ and $x - y - 1 = 0$ are the equations of two internal bisectors of its angles.

20. The equations of the altitudes AD, BE, CF of a triangle ABC are $x + y = 0$, $x - 4y = 0$ and $2x - y = 0$ respectively. Thr coordinates of A are (t, –t). Find coordinates of B and C. Prove that it t varies the locus of the centroid of the triangle ABC is $x + 5y = 0$.

21. For points $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ of the co-ordinate plane, a new distance d(P, Q) is defined by $d(P, Q) = |x_1 - x_2| + |y_1 - y_2|$. Let O = (0, 0) and A = (3, 2). Prove that the set of points in the first quadrant which are equidistant (with respect to the new distance) from O and A consists of the union of a line segment of finite length and an infinite ray. Sketch this set in a labelled diagram. **[IIT -2000, 10]**

22. Let ABC and PQR be any two triangles in the same plane. Assume that the prependiculars from the points A, B, C to the sides QR, RP, PQ respectively are concurrent. Using vector methods or otherwise, prove that the prependiculars from P, Q, R to BC, CA, AB respectively are also concurrent. **[IIT - 2000, 10]**

23. A straight line L through the origin meets the lines $x + y = 1$ and $x + y = 3$ at P and Q respectively. Through P and Q two straight lines L_1 and L_2 are drawn parallel to $2x - y = 5$ and $3x + y = 5$ respectively. Lines L_1 and L_2 intersect at R. Show that the locus of R, as L varies, is a straight line. **[IIT - 2002, 5]**

24. A straight line L with negative slope passes through the point (8, 2) and cuts the positive coordinate axes at points P and Q. Find the absolute minimum value of OP + OQ, as L varies, where O is the origin. **[IIT - 2002, 5]**

25. The area of the triangle formed by the intersection of a line parallel to x-axis and passing through P(h, k) with the lines $y = x$ and $x + y = 2$ is $4h^2$. Find the locus of the point P. **[IIT - 2005, 2]**

ANSWER
EXERCISE–1

Q 1. $90°$ **Q 2.** $\left(\frac{3}{7}, \frac{22}{7}\right)$ **Q 3.** (33, 26) **Q 4.** $K = 7$ or $\frac{31}{9}$

Q 5. 1 : 2 ; Q(–5, –3) **Q 6.** $83x - 35y + 92 = 0$ **Q 7.** $2x + y - 1 = 0$

Q 8. $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(-\frac{3}{2}, \frac{3}{2}\right)$ **Q 10.** $x - y = 0$ **Q 11.** 91 sq.units

Q 12. 6 units **Q 13.** $a^2 + b^2 = a'^2 + b'^2$; $(a + a')x + (b + b')y + (c + c') = 0$; $(a - a')x + (b - b')y = 0$

Q 15. c = – 4 ; B(2 , 0) ; D(4 , 4) **Q 16.** $x + 5y + 5\sqrt{2} = 0$ or $x + 5y - 5\sqrt{2} = 0$

Q 17. $x - 3y - 31 = 0$ or $3x + y + 7 = 0$ **Q 18.** $2x + 2y + \sqrt{2} = 0$ **Q 19.** $-9°, -81°$

Q 20. $5x - 2y - 7 = 0$ **Q.21** $x - 5 = 0$

Q23. $-\frac{3}{2} < \alpha < -1 \cup \frac{1}{2} < \alpha < 1$ **Q 25.** $x + y = 1$; $x + 9y = 1$ **Q 26.** $0 < \theta < \frac{5\pi}{6} - \tan^{-1} 3$

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Q 27. $(-1, 1)$; $(4, 1)$ **Q 28.** $9x + 46y + 83 = 0$ **Q29.** $(1, -2)$, yes $\left(\frac{1}{3}, -\frac{2}{3}\right)$

EXERCISE–2

Q.1 $x + 4y = 4$; $5x + 2y = 8$ **Q 2.** $52x + 89y + 519 = 0$ **Q.4** $(2, 0)$

Q.5 $7x + 24y + 182 = 0$ or $x = -2$ **Q.7** $(0, 0)$ or $\left(0, \frac{5}{2}\right)$ **Q.8** $14x + 23y = 40$

Q.9 $x - 5y = 0$ or $5x + y = 0$, Area = 2.77 sq.units
Q.10 $3x + 6y - 16 = 0$; $8x + 8y + 7 = 0$; $12x + 6y - 11 = 0$
Q.11 $x^2 + 4y^2 + 4xy + 4x - 2y - 1 = 0$ **Q.12** $(a + b)(ax + by) = ab(a + b - 2h)$

Q.13 $\frac{3}{2}$ sq. units, $\left(3, 3, \frac{3}{4}\right)$, isosceles **Q.14** $6x^2 - xy - y^2 - x - 12y - 35 = 0$

Q.15 $2x - y + 3 = 0, 2x + y - 7 = 0, x - 2y - 6 = 0$ **Q.16** $y^2 = a(x - 3a)$

Q.21 $B\left(-\frac{2t}{3}, -\frac{t}{6}\right), C\left(\frac{t}{2}, t\right)$

Q.24 $(y_1^2 - \delta^2)x^2 - 2x_1y_1xy + (x_1^2 - \delta^2)y^2 = 0$ **Q.25** $4(x^2 + y^2) + (4g + 5f)x + (4f - 5g)y = 0$

EXERCISE–3

Q.1 D **Q.2** $(-1, 1)$ & $(6, 6)$
Q.3 $17x - 10y = 0$ **Q.4 (i)** **(a)** C **(b)** D **(c)** A, C, D
Q.5 $x - 3y + 21 = 0$, $x - 3y + 1 = 0$, $3x + y = 12$, $3x + y = 2$ **Q.6** B

Q.7 **(a)** D **(b)** D **Q.8** $(4, 1) \to (2, 3) \to (3, 3) \to \left(0, 3\sqrt{2}\right)$

Q.9 D **Q.10** **(a)** C ; **(b)** B ; **(c)** B ; **(d)** $x - 3y + 5 = 0$
Q.11 A **Q.12** $y = 2x + 1, y = -2x + 1$

EXERCISE–4

1. C **2.** D **3.** C **4.** A **5.** A **6.** D **7.** B

8. A **9.** A **10.** B **11.** B **12.** C **13.** C **14.** C

15. C **16.** B **17.** A **18.** A **19.** A **20.** B **21.** C

22. ABC **23.** BD **24.** ABC **25.** AC **26.** AD

27. CD

EXERCISE–5

2. $a\cos\left(\frac{\alpha - \beta}{2}\right)$ **4.** $11x - 3y + 9 = 0$

6. $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{4}$ **8.** 2 **9.** 2

10. $5/3 \le \beta \le 7/2$

11. $x + 5y + 5\sqrt{2} = 0$ or $x + 5y - 5\sqrt{2} = 0$

12. $x - 3y - 31 = 0$ or $3x + y + 7 = 0$

13. $(x - 4y + 3)(x - y) = 0$
or $x^2 - 5xy + 4y^2 + 3x - 3y = 0$

14. $\left(\frac{3}{2}, 0\right)\left(\frac{9}{4}, 0\right), \left(\frac{3}{2}, \frac{3}{4}\right), \left(\frac{9}{4}, \frac{3}{4}\right)$

15. $x + 4y = 4$; $5x + 2y = 8$

17. $29x - 2y = 31$

18. $3x + 6y - 16 = 0$; $8x + 8y + 7 = 0$; $12x + 6y - 11 = 0$

19. $2x - y + 3 = 0$, $2x + y - 7 = 0$; $x - 2y - 6 = 0$

20. $B\left(-\frac{2t}{3}, -\frac{t}{6}\right), C\left(\frac{t}{2}, t\right)$

24. 18 **25.** $y = 2x + 1$ or $y = -2x + 1$

